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RINGS OF h-DEFORMED DIFFERENTIAL OPERATORS

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We describe the center of the ring $\mathrm{Diff}_{\mathbf{h}}(n)$ *of* **h***-deformed differential operators of type A. We establish an isomorphism between certain localizations of* $\mathrm{Diff}_{\mathbf{h}}(n)$ *and the Weyl algebra* W_n*, extended by* n *indeterminates.*

Keywords: reduction algebra, oscillatory realization, ring of differential operators, Gelfand–Kirillov conjecture, dynamical Yang–Baxter equation

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In memory of Petr Kulish

1. Introduction

The ring $\mathrm{Diff}_{\mathbf{h}}(n)$ of **h**-deformed differential operators of type A appears in the theory of reduction algebras. A reduction algebra $\mathrm{R}^{\mathcal{A}}_{\mathfrak{g}}$ provides a tool for studying decompositions of representations of an associative algebra $\mathcal{A}$ with respect to its subalgebra in the situation where this subalgebra is the universal enveloping algebra of a reductive Lie algebra $\mathfrak{g}$ [1], [2] (see [3], [4] for the definition, general theory, and uses of reduction algebras).

Decompositions of tensor products of representations of a reductive Lie algebra $\mathfrak{g}$ is a particular case of a restriction problem associated with the diagonal embedding of $\mathrm{U}(\mathfrak{g})$ into $\mathrm{U}(\mathfrak{g})\otimes\mathrm{U}(\mathfrak{g})$. The corresponding reduction algebra, denoted by $\mathcal{D}(\mathfrak{g})$, is called a *diagonal reduction algebra* [5]. A description of the diagonal reduction algebra $\mathcal{D}(\mathbf{gl}_n)$ in terms of generators and (ordering) defining relations was given in [5], [6].

The diagonal reduction algebra $\mathcal{D}(\mathbf{gl}_n)$ admits an analogue of the *oscillator realization* in the rings $\mathrm{Diff}_{\mathbf{h}}(n,N)$, $N=1,2,3,\ldots$, of **h**-deformed differential operators (see [7]). The ring $\mathrm{Diff}_{\mathbf{h}}(n,N)$ can be obtained by the reduction of the ring of differential operators in nN variables (i.e., of the Weyl algebra $\mathrm{W}_{nN}=\mathrm{W}_n^{\otimes N}$) with respect to the natural action of $\mathbf{gl}_n$. Similarly to the ring of q-differential operators [8], the algebra $\mathrm{Diff}_{\mathbf{h}}(n,N)$ can be described in the R-matrix formalism. The R-matrix needed here is a solution of the so-called dynamical Yang–Baxter equation (see [9]–[11] for different aspects of the dynamical Yang–Baxter equation and its solutions).

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