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Multidimensional Nonlinear Ion-Acoustic Waves in a Plasma in View of Relativistic Effects[1]

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Abstract—The structure and dynamics of ion-acoustic waves in an unmagnetized plasma, including the case of weakly relativistic collisional plasma (when it is necessary to take into account the high energy particle flows which are observed in the magnetospheric plasma), are studied analytically and numerically on the basis of a model of the Kadomtsev-Petviashvili (KP) equation. It is shown that, if the velocity of plasma particles approaches the speed of light, the relativistic effects start to strongly influence on the wave characteristics, such as its phase velocity, amplitude, and characteristic wavelength, with the propagation of the two-dimensional solitary ion-acoustic wave. The results can be used in the study of nonlinear wave processes in the magnetosphere and in laser and astrophysical plasma.

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1. INTRODUCTION. STATEMENT OF A PROBLEM

Despite the essential progress taking place in this field in recent years (e.g., Belashov and Vladimirov, 2005; McKerr et al., 2014, 2016; and numerous references in these works), the study of nonlinear wave processes in real media with dispersion still remains actual. In particular, this concerns the dynamics of fluctuations in cases in which high energy particle flows in the medium (magnetosphere, compact astrophysical systems, e.g., white dwarfs, laser plasma (Haas, 2014; Shukla et al., 1984)) take place, essentially changing such parameters of propagating wave structures as their phase velocity, amplitude, and characteristic length. A rather large number of works is devoted to investigations of such relativistic effects (e.g., Canuto and Ventura, 1977; Giamarchi, 2003; McKerr et al., 2016; Passoni et al., 2010; Rahman and Ali, 2014; Shukla and Eliasson, 2008); however, practically all of them consider only a one-dimensional (1D) approach. In particular, in (McKerr et al., 2014, 2016) and in earlier works by Washimi and Taniuti (1966) and Das and Paul (1985), the relativistic effects for the ion-acoustic branch of oscillations were investigated in a 1D plasma. The studies by Nejon (1987) and Taniuti and Wei (1968) are perhaps exceptions: however, only some extreme cases were studied in these papers.

The purpose of our work is to study the relativistic effects in the dynamics of ion-acoustic multidimensional nonlinear wave structures in electron-ionic plasma, which is especially important in astrophysical applications and in magnetosphere physics. To solve this problem, in principle, we could start from the general set of hydrodynamic equations for the relativistic case (e.g., Elsässer and Popel, 1997); however, since we are interested in the effects which are displayed at relativistic velocities in comparison with the nonrelativistic case, it would be more logical to consider first the nonrelativistic approach and, further, introducing the relativistic factor (by analogy with (Nejon, 1987)) to consider its influence on the time-space characteristics of multidimensional nonlinear ion-acoustic wave. We shall undertake this approach further.

In the absence of the magnetic field and for a negligible ion temperature, the equations of motion and continuity for ions take the form (Belashov, 1997)

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{e}{M}\nabla\varphi, \quad \frac{\partial n_i}{\partial t} + \operatorname{div}(n_i\mathbf{v}) = 0, \qquad (1)$$

where M is the mass of an ion and φ is the electric potential. Comparison with the equations in generalized variables for an ideal gas in neglect a dissipation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} + \frac{c^2}{\rho}\nabla\rho = 0, \quad \frac{\partial \rho}{\partial t} + \nabla(\rho\mathbf{v}) = 0,$$

where ρ and $c = c(\rho)$ have the sense of a generalized "density" and velocity of "sound," respectively, at density ρ in neglect a dispersion (Karpman, 1973), shows that in this case the ion density n_i and ion-acoustic velocity $c_s = (T_e/M)^{1/2}$ play the role of ρ and c; the

[1] The article was translated by the author.