ISSN 0015-4628, Fluid Dynamics, 2017, Vol. 52, No. 4, pp. 547–560. © Pleiades Publishing, Ltd., 2017.
Original Russian Text © M.M. Alimov, K.G. Kornev, 2017, published in Izvestiya Rossiiskoi Akademii Nauk, Mekhanika Zhidkosti i Gaza, 2017, No. 4, pp. 97–112.

An External Meniscus on a Thin Ovoidal Fiber (The Case of Full Wetting)

M. M. Alimov[a*] **and K. G. Kornev**[b]

[a] *Kazan Federal University, ul. Kremlevskaya 18, Kazan, 420008 Russia*

[b] *Clemson University, Clemson, SC, USA*

e-mail: [*] *Mars.Alimov@kpfu.ru*

Received November 14, 2016

Abstract—A complex shape of an external meniscus formed due to the capillary rise of a liquid along a fiber having the ovoidal profile is considered. Within the framework of the asymptotic approach and under the assumption on the complete wetting of the fiber material by the liquid, an analytical solution of the problem is derived. The particular examples of the meniscus configuration are presented in the cases in which the fiber profile has the shape of an ovoid or an ellipse.

Keywords: capillary rise, minimum surfaces, matching of asymptotic expansions.

DOI: 10.1134/S0015462817040093

When an isolated fiber is immersed into a liquid, the capillary rise of the liquid along the fiber changes the shape of the free surface of the liquid and forms, as a result, an external meniscus. The meniscus configuration is determined by both the fiber profile and the wetting properties of the fiber surface [1–3]. The menisci formed by round fibers are fairly well studied [4–8]. There are also some studies [6, 9–11], in which the numerical and asymptotic methods were applied to investigate the menisci formed by fibers with a smooth, near-circular profile.

In particular, in [11] shape of the meniscus on a fiber of elliptical cross-section was determined using numerical methods. An analysis of the results of that study revealed an important limitation of the computational difference methods: they lose their effectiveness in the case of small contact angles characterizing the wettability of the fiber material. This is attributable to the fact that in the case of small contact angles the level function describing the meniscus shape is characterized by large, and in the case of complete wetting even infinite, gradient values on the fiber surface. This seems an almost undefeatable obstacle in direct numerical simulation of the problem using the difference methods. Thus, in the above-mentioned study [11] the smallest calculated value of the contact angle was about $\pi/3$.

In [12] an asymptotic approach to an analysis of the configuration of an external meniscus on a fiber with an arbitrary profile was developed under the condition of the smallness of its characteristic dimension compared to the capillary length. Below it is shown that in the case of complete wetting the methods developed in [12] can be applied to the derivation of an analytical solution of the problem of the meniscus shape for the fibers, whose profiles belong to a certain class of smooth convex profiles.

1. FORMULATION OF THE PROBLEM

Let as a result of fiber immersion into a liquid there be formed an external meniscus Σ with the contact line Γ_c on the interface of three media, namely, the air, the liquid, and the fiber material (Fig. 1a). The profile of the normal section Γ of the fiber is smooth, convex, and has one axis of symmetry (it is the x axis in Fig. 1b).

We will take half profile thickness along the axis of symmetry for the scale length L_m: $L_m = |BD|/2$. Apart from this length, one more scale can be introduced in the problem; this is the capillary length