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PARTIAL DIFFERENTIAL EQUATIONS

Boundary Value Problems for a Third-Order Hyperbolic Equation on the Plane

E. A. Utkina
Kazan Federal University, Kazan, 420008 Russia
e-mail: eutkina1@yandex.ru

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Abstract—For a factorized third-order hyperbolic equation on the plane, we obtain sufficient conditions for the solvability of some boundary value problems with conditions that have not been considered for this equation earlier.

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The present paper deals with the equation

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)(u_{xy}+au_x+bu_y+cu)=0, \tag{1}$$

which belongs to one of the canonical forms presented in [1]. A number of problems for this equation were studied in [2–8].

Let $D=\{(x,y)\in\mathbb{R}^2:\ 0<x<x_1,\ 0<y<y_1\}\backslash K$, where K is the characteristic $y=x$ of Eq. (1). In addition, set $D_1=\{(0,y)\in\mathbb{R}^2:\ y\in[0,y_1]\}$ and $D_2=\{(x,0)\in\mathbb{R}^2:\ x\in[0,x_1]\}$. We assume that the coefficients of Eq. (1) satisfy the conditions $a\in C^{2,1}(\overline{D})$, $b\in C^{1,2}(\overline{D})$, and $c\in C^{1,1}(\overline{D})$. The class $C^{i,j}$ is defined as the class of functions for which there exist continuous derivatives of order $\partial^{k+l}/\partial x^k\partial y^l$ for any integer $0\le k\le i$ and $0\le l\le j$.

Problem 1. In the domain D, find a function u that is a solution of Eq. (1), belongs to the class $C^{2,1}(D)\cap C^{1,2}(D)\cap C(\overline{D})\cap C^{1,0}(D\cup D_1)\cap C^{0,1}(D\cup D_2)$, and satisfies the conditions

$$u(0,y)=\varphi(y), \tag{2}$$

$$u(x,0)=\psi(x), \tag{3}$$

$$u_x(0,y)=\varphi_1(y), \tag{4}$$

$$u_y(x,0)=\psi_1(x), \tag{5}$$

where $\varphi(0)=\psi(0)$, $\psi,\psi_1\in C^2([0,x_1])$, and $\varphi,\varphi_1\in C^2([0,y_1])$.

Problem 1 was considered earlier in [7, 8], where a sufficient condition for its unique solvability was obtained. In view of the importance of this problem for forthcoming considerations, here we present our proof of this sufficient condition, which is much simpler than the proof given in [7, 8].

First, we represent Eq. (1) in the form $u_{xy}+au_x+bu_y+cu=f$, where f is a solution of the equation $f_x+f_y=0$. Consequently,

$$u_{xy}+au_x+bu_y+cu=\omega(x-y), \tag{6}$$

where $\omega(t)$ is an arbitrary function in the class $C^1([-y_1,x_1])$. We set $\xi=x-y$ and arrive at the problem of finding a solution of the equation

$$u_{xy}+au_x+bu_y+cu=\omega(\xi),\qquad -y_1\le\xi\le x_1, \tag{7}$$

with conditions (2)–(5) and with unknown right-hand side $\omega(\xi)$.