

## UNIVERSAL GENERALIZED COMPUTABLE NUMBERINGS AND HYPERIMMUNITY

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*Generalized computable numberings relative to hyperimmune and high oracles are studied. We give a description of oracles relative to which every finite computable family has a universal computable numbering. Also we present a characterization of the class of oracles relative to which every universal computable numbering of an arbitrary finite family is precomplete, and establish a sufficient condition for universal generalized computable numberings to be precomplete. In addition, we look into the question on limitedness of universal numberings computable relative to high oracles.*

### INTRODUCTION

Generalized computable numberings were introduced and first studied in [1]. A series of questions guiding further research in this area were formulated in [2]. Until recently, investigations of generalized computable numberings were basically concerned with families of sets in the arithmetical and hyperarithmetical hierarchies (see, e.g., [3-5]). A wider approach to studying generalized computable numberings was applied in [6]—namely, instead of jumps of the empty set, we started to consider arbitrary oracles.

Here, use is made of the same approach to studying generalized computable numberings that was employed in [6], and we consider questions on properties of universal numberings computable relative to hyperimmune and high oracles. In formulations of all results of the paper, elements

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