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ON THE GEOMETRY OF SUBMANIFOLDS IN E^n_{2n}

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Abstract

A special class of $2m$-dimensional submanifolds in a $2n$-dimensional pseudo-Euclidean space with metric of signature (n, n), known as a pseudo-Euclidean Rashevsky space, is studied. For such submanifolds, canonical integrals and parametric equations are found.

Key words: even-dimensional submanifolds, pseudo-Euclidean Rashevsky space, double fiber bundle, canonical integral, differential-geometric structure, fibration, foliation.

One of the most characteristic features of modern Differential Geometry is the active application of its methods in the adjoining fields of the mathematical science. Essentially increased effectiveness of these methods which accumulated fundamental achievements, first of all from the general algebra and theory of differential equations, in combination with the tendency to consider various mathematical objects as differential-geometric structures on corresponding manifolds, has led, on the one hand, to the appearance of new directions of the differential geometric study, and, on the other hand, to a more fundamental, geometric interpretation of these objects.

The next step is the differential geometric analysis of these structures and identification of their most general characteristic (geometric) properties. Finally, on the last stage of research, these properties or their part become the foundation for generalizations and new problems in the initial theory.

Moreover, in accordance with [1], they assume in particular the exact description of the category of the structures under study and also the identification of the category of algebraic systems necessary for their study.

All above mentioned is true for the geometry of multiple integral depending on parameters. The study of differential geometric structures defined by such an integral on the manifold of integration variables and parameters to a certain extent is similar to the study of the integral geometry [2–4]. At the same time the presence of parameters totally changes the cycle of arising problems and corresponding results. By systematic study of multiple integrals depending on parameters (in a special case when the number of parameters is equal to the number of variables) and corresponding integral transforms one can see a good number of interesting geometrical problems connected with the description of invariant properties of such integrals.

The present article is devoted to the study of a special class of $2m$-dimensional submanifolds with structure of double fiber bundle in the $2n$-dimensional pseudo-Euclidean space E^n_{2n} with metric of index n. We find multiple integral depending on parameters, determining the structure of such a submanifold on the corresponding manifold of integration variables and parameters, also parametric equations of this submanifold.

1. Pseudo-Riemannian Rashevsky space

In 1925, Russian geometer P.A. Shirokov from Kazan State University introduced [5] the special class of even-dimensional symmetric spaces known as A-spaces or elliptic A-spaces. In 1933, E. Kähler [6] studied the same spaces known now as Kähler spaces.

Let us consider a $2n$-dimensional manifold M with local coordinates $x^1, \dots, x^n$, $y_1, \dots, y_n$ such that in all admissible transformations of coordinates two sets of n coordinates are separated: the transformed coordinates $x^1, \dots, x^n$ are functions of $x^1, \dots, x^n$ and the same is true for the second set of coordinates. Consider a real kern function $U(x^1, \dots, x^n, y_1, \dots, y_n)$ and introduce the following values

$$g^i_j = \frac{\partial^2 U}{\partial x^j \partial y_i}.$$

It is easy to check that the matrix

$$G = \begin{pmatrix} 0 & g^i_j \\ g^j_i & 0 \end{pmatrix}$$

is invariant under all admissible transformations of local coordinates. This matrix is nondegenerate and, therefore, its elements introduce a metric on M. In its turn this metric generates an affine connection on M. On this manifold, the fibers from different families are complex conjugate.

The so called hyperbolic case when both the families of fibers are real n-dimensional manifolds was introduced by P.K. Rashevsky [7]. He studied an invariant scalar field $U(x^1, \dots, x^n, y_1, \dots, y_n)$ with nondegenerate matrix of second order derivatives:

$$\det\left(\frac{\partial^2 U}{\partial x^j \partial y_i}\right) \neq 0$$

and, using this matrix, introduced a pseudo-Riemannian metric of index n on M and the corresponding pseudo-Riemannian connection. This space is known as a Rashevsky pseudo-Riemannian space. It has the following characteristic properties.

1. The scalar field $U(x^1, \dots, x^n, y_1, \dots, y_n)$ generating the structure of a pseudo-Riemannian space on M is determined with arbitrariness

$$U(x^i, y_j) \longrightarrow U(x^i, y_j) + U_1(x^i) + U_2(y_j).$$

2. Each point of M belongs to one and only one fiber from each of the two families of fibers. Fibers from different families have intersection in no more than one point.

3. The fibers of both the families are isotropic.

4. The fibers of each family have the property of absolute parallelism (auto parallelism): vectors tangent to fibers from one of the families remain tangent to them after parallel transfer along an arbitrary smooth curve.

It follows from each of the two latter properties that both the families of fibers are totally geodesic in M.

This space was studied by P.K. Rashevsky and other researchers as an example of a pseudo-Riemannian space only, without any relations to other fields of Mathematics and Physics.

Later, in 60-th, professor V.V. Vishnevsky from Kazan State University introduced the third type of A-spaces (parabolic A-spaces) [8] and completed the classification of these structures.

In terms of a co-basis of linear differential forms $\omega^1, \dots, \omega^n, \omega_1, \dots, \omega_n$ adapted to the structure of $2n$-dimensional Rashevsky space, the structure equations of this space can be presented in the form [9]

$$\begin{aligned} &d\omega^I = \omega^I_K \wedge \omega^K, \\ &d\omega_I = -\omega^K_I \wedge \omega_K, \qquad\qquad I, K, P, T = 1, \ldots, n \\ &d\omega^I_K = \omega^I_P \wedge \omega^P_K + R^{IT}_{KP} \omega^P \wedge \omega_T, \end{aligned} \tag{1.1}$$

where R^{IT}_{KP} are the nonzero components of the curvature tensor. The metric of this space is generated by the nondegenerate bilinear closed form [9]

$$d\varphi = \omega^I \wedge \omega_I\ . \tag{1.2}$$

It is known [9] that an integral of the form $\lambda\omega^1 \wedge \ldots \wedge \omega^n$ induces a structure of a $2n$-dimensional Rashevsky space on the manifold M of integration variables and parameters under natural condition of nondegeneracy for the matrix of second order derivatives of the function $\ln \lambda$.

Rashevsky space can also be considered as a double fibered manifold with two families of n-dimensional transverse geodesic fibers. It is a generalization of the cross product of two manifolds: a $(2n + s)$-dimensional smooth manifold M is said to be a double fiber bundle if two smooth mappings

$$\pi_i : M \longrightarrow M_i, \quad i = 1, 2,$$

from M onto n- and $n + s$-dimensional smooth manifolds M_1 and M_2 are given, the fibers, i. e., full preimages of points from M_1 and M_2 under the mappings π_1 and π_2 respectively are smooth $n + s$- and n-dimensional submanifolds, and the tangent spaces to the fibers of the bundles π_1 and π_2 at an arbitrary point have only trivial intersection:

$$T_P\left(\pi_1^{-1}(x)\right) \cap T_P\left(\pi_2^{-1}(y)\right) = p, \quad \pi_1(p) = x,\ \pi_2(p) = y, \quad x \in M_1,\ y \in M_2.$$

Therefore, the tangent space of M at an arbitrary point is a direct sum of $n + s$- and n-dimensional subspaces. The case of Rashevsky spaces corresponds to $s = 0$.

If the curvature tensor of such a space is trivial, we have a pseudo-Euclidean Rashevsky space which, in terms of a co-frame of principal exterior linear differential forms $\omega^1, \ldots, \omega^n, \omega_1, \ldots, \omega_n$ adapted to the structure of a double fiber bundle on E^n_{2n} and defined on the principal fiber bundle of tangent frames on E^n_{2n}, can be presented by the following structure equations [9]

$$\begin{aligned} &d\omega^I = \omega^I_K \wedge \omega^K, \\ &d\omega_I = -\omega^K_I \wedge \omega_K, \qquad I, K, P = 1, \ldots, n \\ &d\omega^I_K = \omega^I_P \wedge \omega^P_K, \end{aligned} \tag{1.3}$$

where the secondary forms ω^I_K are defined on the manifold $T^2E^n_{2n}$ of second order tangent frames associated to E^n_{2n} and do not depend on the principal forms. There is a natural connection between such spaces and the Fourier transform [9]: this integral transform invariantly induces a structure of E^n_{2n} on the double fibered manifold of integration variables and parameters M.

An $(n + s)$-tuple integral depending on n parameters is said to be a canonical integral of a differential-geometric structure on a $2n + s$-dimensional manifold M if this integral generates the structure on M. Suppose that n-tuple integral depending on n parameters generates a structure of a Rashevsky $2n$-dimensional space on a manifold of integration variables and parameters. An n-tuple integral depending on n parameters constructed on parameters of integration generates the same structure of a Rashevsky $2n$-dimensional space on the manifold of integration variables and parameters M if and only if M is an Einstein space (Ricci tensor is proportional to the metric tensor with constant coefficient). This is the geometrical meaning of the invertibility of the corresponding integral transform. This means that the category of Einstein $2n$-dimensional spaces with metric of signature (n, n) is the most general one for the construction of invertible integral transforms. One of the most important problems here is findind of an integral transform generating the structure of a given Rashevsky (Einstein) space on M.

2. 2m-dimensional submanifolds with structure of double fiber bundle in a pseudo-Euclidean space $\mathbf{E^n_{2n}}$

The necessity to consider submanifolds of the pseudo-Euclidean spaces E^n_{2n} and corresponding canonical integrals follows from the problem of finding canonical integrals of Rashevsky (Einstein) spaces because in the special case when the space under study is pseudo-Euclidean (the curvature is equal to zero) the corresponding canonical integral coincides with the classical Fourier transform. Taking into account that an integral generates the corresponding differential-geometric structure in an invariant way and that the geometry of a pseudo-Riemannian space, in general, is defined by its curvature tensor, it is natural to suppose that the canonical integral of a Rashevsky (Einstein) space is related to the curvature tensor of this space in a special way.

Let us consider $2m$-dimensional submanifold M with structure of a double fiber bundle in an $2n$-dimensional pseudo-Euclidean space E^n_{2n} with metric of index n (pseudo-Euclidean Rashevsky space) when the dimension n satisfies the condition $2m > n$.

Suppose that, in terms of a co-basis of linear differential forms ω^1, $\omega^2, \ldots, \omega^n$, ω_1, $\omega_2, \ldots, \omega_n$ adapted to the structure of a pseudo-Euclidean Rashevsky space E^n_{2n}, the structure equations of the space are represented in the form (1.3) and that a $2m$-dimensional submanifold M is defined by the equations

$$\omega^{m+i} = \omega_{2m-n+i}, \quad \omega_{m+i} = \omega^i, \quad i = 1, \ldots n - m. \tag{2.1}$$

Let us note that relations (2.1) determine the most general class of $2m$-dimensional submanifolds in E^n_{2n} with structure of a double fiber bundle. This class is a direct generalization of the corresponding classes of submanifolds of codimension two, studied in [10, 11].

There are three possible cases: 1) $2(n-m) > m$ or $3m < 2n$, 2) $2(n-m) = m$ or $3m = 2n$, 3) $2(n-m) < m$ or $3m > 2n$.

The case 3) was studied in [12]. Let us study the case $3m < 2n$. Therefore the following inequalities hold

$$3n < 6m < 4n.$$

This condition is equivalent to the inequality $2m - n < n - m$. Let us introduce new indices $\alpha = 1, \ldots, 2m-n$; $\xi = 2m-n+1, \ldots, n-m$; $a = n-m+1, \ldots, m$. The metric form of the total Rashevsky space E^n_{2n} defined by the invariant bilinear closed nondegenerate form $d\varphi = \omega^I \wedge \omega_I$ induces the bilinear form

$$d\varphi^* = \omega^\alpha \wedge \omega_\alpha + \omega^\xi \wedge \omega_\xi + \omega^a \wedge \omega_a + \delta^\xi_{2m-n+\alpha} \omega_\xi \wedge \omega^\alpha + \delta^a_{2m-n+\xi} \omega_a \wedge \omega^\xi \tag{2.2}$$

on M.

Substitution of relations (2.1) into (1.3) and application of the above introduced indices gives the following general structure equations of a submanifold M

$$
\begin{aligned}
d\omega^\alpha &= \omega^\alpha_\beta \wedge \omega^\beta + \omega^\alpha_\xi \wedge \omega^\xi + \omega^\alpha_a \wedge \omega^a + \omega^\alpha_{m+k} \wedge \omega_{2m-n+k},\\
d\omega^\xi &= \omega^\xi_\eta \wedge \omega^\eta + \omega^\xi_\alpha \wedge \omega^\alpha + \omega^\xi_a \wedge \omega^a + \omega^\xi_{m+k} \wedge \omega_{2m-n+k},\\
d\omega^a &= \omega^a_b \wedge \omega^b + \omega^a_\alpha \wedge \omega^\alpha + \omega^a_\xi \wedge \omega^\xi + \omega^a_{m+k} \wedge \omega_{2m-n+k},\\
d\omega_\alpha &= -\omega^\beta_\alpha \wedge \omega_\beta - \omega^\xi_\alpha \wedge \omega_\xi - \omega^a_\alpha \wedge \omega_a - \omega^{m+k}_\alpha \wedge \omega^k,\\
d\omega_\xi &= -\omega^\eta_\xi \wedge \omega_\eta - \omega^\alpha_\xi \wedge \omega_\alpha - \omega^a_\xi \wedge \omega_a - \omega^{m+k}_\xi \wedge \omega^k,\\
d\omega_a &= -\omega^b_a \wedge \omega_b + \omega^\alpha_a \wedge \omega_\alpha - \omega^\xi_a \wedge \omega_\xi - \omega^{m+k}_a \wedge \omega^k,\\
d\omega^\alpha_\beta &= \omega^\alpha_\gamma \wedge \omega^\gamma_\beta + \omega^\alpha_\xi \wedge \omega^\xi_\beta + \omega^\alpha_a \wedge \omega^a_\beta + \omega^\alpha_{m+k} \wedge \omega^{m+k}_\beta,\\
d\omega^\xi_\eta &= \omega^\xi_\mu \wedge \omega^\mu_\eta + \omega^\xi_\alpha \wedge \omega^\alpha_\eta + \omega^\xi_a \wedge \omega^a_\eta + \omega^\xi_{m+k} \wedge \omega^{m+k}_\eta,\\
d\omega^a_b &= \omega^a_c \wedge \omega^c_b + \omega^a_\alpha \wedge \omega^\alpha_b + \omega^a_\xi \wedge \omega^\xi_b + \omega^a_{m+k} \wedge \omega^{m+k}_b,\\
d\omega^\alpha_\xi &= \omega^\alpha_\beta \wedge \omega^\beta_\xi + \omega^\alpha_\eta \wedge \omega^\eta_\xi + \omega^\alpha_a \wedge \omega^a_\xi + \omega^\alpha_{m+k} \wedge \omega^{m+k}_\xi,\\
d\omega^\alpha_a &= \omega^\alpha_\beta \wedge \omega^\beta_a + \omega^\xi_\alpha \wedge \omega^\xi_a + \omega^a_c \wedge \omega^c_a + \omega^\alpha_{m+k} \wedge \omega^{m+k}_a,\\
d\omega^\xi_\alpha &= \omega^\xi_\beta \wedge \omega^\beta_\alpha + \omega^\xi_\eta \wedge \omega^\eta_\alpha + \omega^\xi_\alpha \wedge \omega^a_\alpha + \omega^\xi_{m+k} \wedge \omega^{m+k}_\alpha,\\
d\omega^\xi_a &= \omega^\xi_\alpha \wedge \omega^\alpha_a + \omega^\xi_\eta \wedge \omega^\eta_a + \omega^\xi_b \wedge \omega^b_a + \omega^\xi_{m+k} \wedge \omega^{m+k}_a,\\
d\omega^a_\alpha &= \omega^a_\beta \wedge \omega^\beta_\alpha + \omega^a_\xi \wedge \omega^\xi_\alpha + \omega^a_b \wedge \omega^b_\alpha + \omega^a_{m+k} \wedge \omega^{m+k}_\alpha,\\
d\omega^a_\xi &= \omega^a_\alpha \wedge \omega^\alpha_\xi + \omega^a_\eta \wedge \omega^\eta_\xi + \omega^a_b \wedge \omega^b_\xi + \omega^a_{m+i} \wedge \omega^{m+i}_\xi,
\end{aligned}
\tag{2.3}
$$

where the secondary forms ω^α_β, ω^ξ_η, ω^a_b, ω^α_ξ, ω^α_a, ω^ξ_α, ω^ξ_a, ω^a_α, ω^a_ξ and ω^α_{m+k}, ω^ξ_{m+k}, ω^a_{m+k}, ω^{m+k}_α, ω^{m+k}_ξ, ω^{m+k}_a are defined on the manifold $T^{(2)}M$ of second order tangent frames associated to the manifold M and adapted to its structure.

Taking into account that the bilinear form $d\varphi^*$ is closed and using exterior differentiation of (2.2) with application of general structure equations (2.3), we arrive at the identity which shows that, in the general case, the forms ω^α_ξ, ω^ξ_a, $\delta^\xi_{2m-n+\beta}\omega^\beta_\alpha - \delta^\eta_{2m-n+\alpha}\omega^\xi_\eta$, $\delta^a_{2m-n+\xi}\omega^\xi_\alpha - \delta^\xi_{2m-n+\alpha}\omega^a_\xi$, $\delta^\xi_{2m-n+\alpha}\omega^\alpha_\eta - \delta^a_{2m-n+\eta}\omega^\xi_a$, $\delta^a_{2m-n+\xi}\omega^\xi_\eta - \delta^b_{2m-n+\eta}\omega^a_b$ are principal. We will use these general conditions for more detailed research of the differential-geometric structure on M.

Taking into account that the submanifold M has a structure of a double fiber bundle, i. e., that the following systems of linear differential equations

$$
\begin{aligned}
&\omega^\alpha = 0, \quad \omega^\xi = 0, \quad \omega^a = 0,\\
&\qquad \alpha = 1, \dots, 2m-n, \quad \xi = 2m-n+1, \dots, n-m, \quad a = n-m+1, \dots, m;\\
&\omega_\alpha = 0, \quad \omega_\xi = 0, \quad \omega_a = 0,\\
&\qquad \alpha = 1, \dots, 2m-n, \quad \xi = 2m-n+1, \dots, n-m, \quad a = n-m+1, \dots, m
\end{aligned}
$$

are totally integrable, we arrive at the following system of identities

$$
\begin{aligned}
&\omega^\alpha_{m+k} \wedge \omega_{2m-n+k} = 0, \quad \omega^\xi_{m+k} \wedge \omega_{2m-n+k} = 0, \quad \omega^a_{m+k} \wedge \omega_{2m-n+k} = 0,\\
&\omega^{m+k}_\alpha \wedge \omega^k = 0, \quad \omega^{m+k}_\xi \wedge \omega^k = 0, \quad \omega^{m+k}_a \wedge \omega^k = 0.
\end{aligned}
\tag{2.4}
$$

Applying Cartan's lemma [1], one can easily see that the secondary forms ω^{α}_{m+i}, ω^{ξ}_{m+i}, ω^{a}_{m+i} and ω^{m+i}_{α}, ω^{m+i}_{ξ}, ω^{m+i}_{a} are linear combinations of the basic linear differential forms ω_ξ, ω_a, $\xi = 2m-n+1,\dots,n-m$, $a = n-m+1,\dots,m$ and ω^α, ω^ξ, $\alpha = 1,\dots,2m-n$, $\xi = 2m-n+1,\dots,n-m$ respectively.

On the other hand, the exterior differentiation of relations (2.1), which are identities on M, gives the following two identities

$$\left(\omega^{m+i}_{k} + \omega^{m+k}_{2m-n+i}\right) \wedge \omega^k + \omega^{m+i}_{a} \wedge \omega^a + \omega^{\alpha}_{2m-n+i} \wedge \omega_\alpha + \left(\omega^{m+i}_{n-m+\xi} + \omega^{\xi}_{2m-n+i}\right) \wedge \omega_\xi + \left(\omega^{m+i}_{n-m+a} + \omega^{a}_{2m-n+i}\right) \wedge \omega_a = 0,$$

$$\left(\omega^{m+k}_{m+i} + \omega^{i}_{k}\right) \wedge \omega^k + \omega^{i}_{a} \wedge \omega^a + \omega^{\alpha}_{m+i} \wedge \omega_\alpha + \left(\omega^{i}_{n-m+\xi} + \omega^{\xi}_{m+i}\right) \wedge \omega_\xi + \left(\omega^{i}_{n-m+a} + \omega^{a}_{m+i}\right) \wedge \omega_a = 0.$$

Taking into account identities (2.4), it is easy to check that this system is equivalent to the system of the following four identities

$$\begin{aligned}
&\left(\omega^{m+i}_{k} + \omega^{m+k}_{2m-n+i}\right) \wedge \omega^k + \omega^{m+i}_{a} \wedge \omega^a = 0,\\
&\omega^{\alpha}_{2m-n+i} \wedge \omega_\alpha + \left(\omega^{m+i}_{n-m+\xi} + \omega^{\xi}_{2m-n+i}\right) \wedge \omega_\xi + \left(\omega^{m+i}_{n-m+a} + \omega^{a}_{2m-n+i}\right) \wedge \omega_a = 0,\\
&\left(\omega^{m+k}_{m+i} + \omega^{i}_{k}\right) \wedge \omega^k + \omega^{i}_{a} \wedge \omega^a = 0,\\
&\omega^{\alpha}_{m+i} \wedge \omega_\alpha + \left(\omega^{i}_{n-m+\xi} + \omega^{\xi}_{m+i}\right) \wedge \omega_\xi + \left(\omega^{i}_{n-m+a} + \omega^{a}_{m+i}\right) \wedge \omega_a = 0.
\end{aligned} \tag{2.5}$$

It follows directly from the obtained system that all the secondary forms ω^{α}_{a} are equal to zero identically. Indeed it's follows from the third identity from (2.5) that the secondary forms ω^{α}_{a} are linear combinations of the basic principal differential forms ω^1, $\omega^2,\dots,\omega^n$. But it is easy to see from the second identity of system (2.5) that the same forms have expansions in terms of the basic principal differential forms $\omega_1,\ \omega_2,\dots,\omega_n$ only. This is possible if and only if the forms ω^{α}_{a} are equal to zero.

Besides the first identity of system (2.4) shows that the secondary forms ω^{α}_{m+i} have nontrivial expansions in terms of the basic principal forms $\omega_{2m-n+1},\dots,\omega_n$ only. Substituting the corresponding expansions into the fourth identity of system (2.5), we see that the secondary forms ω^{α}_{m+i} are vanishing.

Let us note now that, as follows from the last identity of system (2.4), the secondary forms ω^{m+i}_{a} are linear combinations of the basic principal forms $\omega^1,\omega^2,\dots,\omega^{n-m}$. Substitution of the corresponding expansions into the first identity of system (2.5) shows that all the forms ω^{m+i}_{a} are vanishing too.

Using relations (2.4), it is easy to check that system of identities (2.5) is equivalent to the following system

$$\begin{aligned}
&\omega^{n-m+a}_{k} \wedge \omega^k = 0, \quad \left(\omega^{n-m+\xi}_{k} + \omega^{m+k}_{\xi}\right) \wedge \omega^k = 0,\\
&\omega^{\alpha}_{\xi} \wedge \omega_\alpha + \left(\omega^{n-m+\xi}_{n-m+\eta} + \omega^{\eta}_{\xi}\right) \wedge \omega_\eta = 0,\\
&\left(\omega^{n-m+a}_{n-m+\xi} + \omega^{\xi}_{a}\right) \wedge \omega_\xi = 0, \quad \omega^{\xi}_{m+\alpha} \wedge \omega_\xi + \omega^{a}_{m+\alpha} \wedge \omega_a = 0,\\
&\left(\omega^{m+k}_{m+\xi} + \omega^{\xi}_{k}\right) \wedge \omega^k + \omega^{\xi}_{a} \wedge \omega^a = 0,\\
&\left(\omega^{\xi}_{n-m+\eta} + \omega^{\eta}_{m+\xi}\right) \wedge \omega_\eta + \left(\omega^{\xi}_{n-m+a} + \omega^{a}_{m+\xi}\right) \wedge \omega_a = 0.
\end{aligned} \tag{2.5$'$}$$

Exterior differentiation of relation (2.2) shows that the secondary forms ω_a^ξ, ω_ξ^α are principal forms. The application of the second and third identities of system (2.5′) gives the following expansions

$$\begin{aligned} \omega_\xi^\alpha &= C_\xi^{\alpha\beta}\omega_\beta + C_\xi^{\alpha\eta}\omega_\eta, \\ \omega_a^\xi &= C_{a\alpha}^\xi\omega^\alpha + C_{\alpha\eta}^\xi\omega^\eta + C_{ab}^\xi\omega^b \end{aligned} \tag{2.6}$$

with the symmetry conditions on the coefficients corresponding to Cartan's lemma: $C_\xi^{\alpha\beta} = C_\xi^{\beta\alpha}$, $C_{ab}^\xi = C_{ba}^\xi$.

Applying Cartan's lemma [1] to identities of the first and the last identities of system (2.5), we obtain the following expansions

$$\begin{aligned} \omega_\alpha^{m+i} &= C_{\alpha\beta}^{m+i}\omega^\beta + C_{\alpha\xi}^{m+i}\omega^\xi, \qquad & C_{\alpha\beta}^{m+i} &= C_{\beta\alpha}^{m+i}, \\ \omega_\xi^{m+i} &= C_{\alpha\xi}^{m+i}\omega^\alpha + C_{\xi\eta}^{m+i}\omega^\eta, \qquad & C_{\xi\eta}^{m+i} &= C_{\eta\xi}^{m+i}, \\ \omega_{m+i}^\xi &= C_{m+i}^{\xi\eta}\omega_\eta + C_{m+i}^{\xi a}\omega_a, \qquad & C_{m+i}^{\xi\eta} &= C_{m+i}^{\eta\xi}, \\ \omega_{m+i}^a &= C_{m+i}^{\xi a}\omega_\xi + C_{m+i}^{ab}\omega_b, \qquad & C_{m+i}^{ab} &= C_{m+i}^{ba}. \end{aligned} \tag{2.7}$$

Exterior differentiation of expansions (2.6), (2.7) with further application of the general structure equations of a submanifold M gives the following differential identities

$$\begin{gathered}
\left(dC_\xi^{\alpha\beta} - C_\xi^{\alpha\gamma}\omega_\gamma^\beta - C_\xi^{\gamma\beta}\omega_\gamma^\alpha + C_\eta^{\alpha\beta}\omega_\xi^\eta\right)\wedge\omega_\beta + \\
+\left(dC_\xi^{\alpha\eta} - C_\xi^{\alpha\nu}\omega_\nu^\eta - C_\xi^{\beta\eta}\omega_\beta^\alpha + C_\nu^{\alpha\eta}\omega_\xi^\nu - C_\xi^{\alpha\beta}\omega_\beta^\eta\right)\wedge\omega_\eta - C_\xi^{\alpha\beta}\omega_\beta^a\wedge\omega_a - C_\xi^{\alpha\eta}\omega_\eta^a\wedge\omega_a = 0, \\
\left(dC_{ab}^\xi + C_{ac}^\xi\omega_b^c + C_{cb}^\xi\omega_a^c - C_{ab}^\eta\omega_\eta^\xi + C_{a\alpha}^\xi\omega_b^\alpha + C_{a\eta}^\xi\omega_b^\eta\right)\wedge\omega^b + \\
+\left(dC_{a\alpha}^\xi + C_{a\beta}^\xi\omega_\alpha^\beta + C_{b\alpha}^\xi\omega_a^b - C_{a\alpha}^\eta\omega_\eta^\xi + C_{ab}^\xi\omega_\alpha^b\right)\wedge\omega^\alpha + \\
+\left(dC_{a\eta}^\xi + C_{a\mu}^\xi\omega_\eta^\mu + C_{b\eta}^\xi\omega_a^b - C_{a\eta}^\mu\omega_\mu^\xi + C_{ab}^\xi\omega_\eta^b\right)\wedge\omega^\eta = 0, \\
\left(dC_{\alpha\beta}^{m+i} + C_{\alpha\gamma}^{m+i}\omega_\beta^\gamma + C_{\gamma\beta}^{m+i}\omega_\alpha^\gamma - C_{\alpha\beta}^{m+k}\omega_{m+k}^{m+i} + C_{\alpha\xi}^{m+i}\omega_\beta^\xi + C_{\beta\xi}^{m+i}\omega_\alpha^\xi\right)\wedge\omega^\beta + \\
+\left(dC_{\alpha\xi}^{m+i} + C_{\alpha\eta}^{m+i}\omega_\xi^\eta + C_{\beta\xi}^{m+i}\omega_\alpha^\beta - C_{\alpha\xi}^{m+k}\omega_{m+k}^{m+i} + C_{\xi\eta}^{m+i}\omega_\alpha^\eta + C_{\alpha\beta}^{m+i}C_\xi^{\gamma\beta}\omega_\gamma\right)\wedge\omega^\xi = 0, \\
\left(dC_{\alpha\xi}^{m+i} + C_{\alpha\eta}^{m+i}\omega_\xi^\eta + C_{\beta\xi}^{m+i}\omega_\alpha^\beta - C_{\alpha\xi}^{m+k}\omega_{m+k}^{m+i} + C_{\xi\eta}^{m+i}\omega_\alpha^\eta + C_{\alpha\beta}^{m+i}C_\xi^{\gamma\beta}\omega_\gamma\right)\wedge\omega^\alpha + \\
+\left(dC_{\xi\eta}^{m+i} + C_{\xi\mu}^{m+i}\omega_\eta^\mu + C_{\mu\eta}^{m+i}\omega_\xi^\mu - C_{\xi\eta}^{m+k}\omega_{m+k}^{m+i} + C_{\alpha\eta}^{m+i}C_\xi^{\alpha\gamma}\omega_\gamma + C_{\alpha\eta}^{m+i}C_\xi^{\alpha\gamma}\omega_\gamma\right)\wedge\omega^\eta = 0, \\
\left(dC_{m+i}^{\xi\eta} - C_{m+i}^{\xi\alpha}\omega_\alpha^\eta - C_{m+i}^{\mu\eta}\omega_\mu^\xi + C_{m+k}^{\xi\eta}\omega_{m+i}^{m+k} - C_{m+i}^{\xi a}C_{ab}^\eta\omega^b - C_{ab}^\xi C_{m+i}^{\eta a}\omega^b\right)\wedge\omega_\eta + \\
+\left(dC_{m+i}^{\xi a} - C_{m+i}^{\xi b}\omega_b^a - C_{m+i}^{\eta a}\omega_\eta^\xi + C_{m+k}^{\xi a}\omega_{m+i}^{m+k} - C_{m+i}^{\xi\eta}\omega_\eta^a - C_{cb}^\xi C_{m+i}^{ca}\omega^b\right)\wedge\omega_a = 0, \\
\left(dC_{m+i}^{\xi a} - C_{m+i}^{\xi b}\omega_b^a - C_{m+i}^{\eta a}\omega_\eta^\xi + C_{m+k}^{\xi a}\omega_{m+i}^{m+k} - C_{m+i}^{\xi\eta}\omega_\eta^a - C_{cb}^\xi C_{m+i}^{ca}\omega^b\right)\wedge\omega_\xi + \\
+\left(dC_{m+i}^{ab} - C_{m+i}^{ac}\omega_c^b - C_{m+i}^{cb}\omega_c^a + C_{m+k}^{ab}\omega_{m+i}^{m+k} - C_{m+i}^{\xi a}\omega_\xi^b - C_{m+i}^{\xi b}\omega_\xi^a\right)\wedge\omega_b = 0.
\end{gathered} \tag{2.8}$$

It follows from first two identities of this system that quantities C^{ξ}_{ab} and $C^{\alpha\beta}_{\xi}$ are invariants and therefore their vanishing has an invariant geometric meaning. For example, if $C^{\alpha\beta}_{\xi} = 0$, then the system of linear differential equations $\omega^{\alpha} = 0$, $\alpha = 1, \dots, 2m - n$ is totally integrable; the condition $C^{\xi}_{ab} = 0$ characterizes the total integrability of the system of Pfaff equations $\omega_a = 0$, $a = n - m + 1, \dots, m$.

Next identities of the system (2.8) show that the quantities $C^{m+i}_{\xi\eta}$, $C^{\xi\eta}_{m+i}$, $C^{m+i}_{\alpha\beta}$, C^{ab}_{m+i} are invariants, and the other quantities occurring in this system are not invariants. Therefore, without any loss of generality, the quantities $C^{m+i}_{\alpha\xi}$, $C^{\xi a}_{m+i}$ can be considered equal to zero.

There are three possible cases:

a) $2m - n < 2n - 3m$, i.e., $5m < 3n$, therefore $15n < 30m < 18n$,

b) $2m - n = 2n - 3m$, i.e., $5m = 3n$, therefore $15n < 30m = 18n$,

c) $2m - n > 2n - 3m$, i.e., $5m > 3n$, therefore $18n < 30m < 20n$.

Let us consider the case $15n < 30m = 18n$. We note that, by virtue of the fact that the ranges of the indices $\alpha = 1, \dots, 2m - n$; $\xi = 2m - n + 1, \dots, n - m$; $a = n - m + 1, \dots, m$ are of the same length, the dimension m is divisible by 3.

Exterior differentiation of identities $\omega^{\alpha}_{a} = 0$, $\omega^{\alpha}_{m+i} = 0$, $\omega^{m+i}_{a} = 0$ and application of the general structure equations of a submanifold M gives the system of relations

$$\omega^{\alpha}_{\xi} \wedge \omega^{\xi}_{a} = 0, \quad \omega^{m+i}_{\xi} \wedge \omega^{\xi}_{a} = 0, \quad \omega^{\alpha}_{\xi} \wedge \omega^{\xi}_{m+i} = 0,$$

and, therefore, by virtue of expansions (2.6) and (2.7), the following system of algebraic relations holds:

$$\begin{gathered}
C^{\alpha\beta}_{\xi} C^{\xi}_{a\gamma} = 0, \quad C^{\alpha\beta}_{\xi} C^{\xi}_{ab} = 0, \quad C^{ab}_{\xi} C^{\xi}_{a\gamma} = 0, \quad C^{\alpha\eta}_{\xi} C^{\xi}_{a\gamma} = 0, \quad C^{\alpha\beta}_{\xi} C^{\xi}_{a\eta} = 0, \\
C^{\alpha\eta}_{\xi} C^{\xi}_{ab} = 0, \quad C^{\alpha\eta}_{\xi} C^{\xi}_{a\mu} = 0, \quad C^{ab}_{\xi} C^{\xi}_{a\eta} = 0, \quad C^{\alpha c}_{\xi} C^{\xi}_{ab} = 0, \\
C^{m+i}_{\alpha\xi} C^{\xi}_{ab} = 0, \quad C^{m+i}_{\xi\eta} C^{\xi}_{ab} = 0, \quad C^{m+i}_{\alpha\xi} C^{\xi}_{a\beta} = C^{m+i}_{\beta\xi} C^{\xi}_{a\alpha}, \\
C^{m+i}_{\xi\eta} C^{\xi}_{a\alpha} = C^{m+i}_{\alpha\xi} C^{\xi}_{a\eta}, \quad C^{m+i}_{\xi\eta} C^{\xi}_{a\mu} = C^{m+i}_{\xi\mu} C^{\xi}_{a\eta}, \\
C^{\alpha\beta}_{\xi} C^{\xi\eta}_{m+i} = 0, \quad C^{\alpha\beta}_{\xi} C^{\xi a}_{m+i} = 0, \quad C^{\alpha\nu}_{\xi} C^{\xi\eta}_{m+i} = C^{\alpha\eta}_{\xi} C^{\xi\nu}_{m+i}, \\
C^{\alpha a}_{\xi} C^{\xi\eta}_{m+i} = C^{\alpha\eta}_{\xi} C^{\xi a}_{m+i}, \quad C^{\alpha b}_{\xi} C^{\xi a}_{m+i} = C^{\alpha a}_{\xi} C^{\xi b}_{m+i}.
\end{gathered} \tag{2.9}$$

It follows from identities (2.5′) that the form $\omega^{m+\xi}_{m+\alpha}$ is principal and that it has an expansion in terms of the principal forms ω_{α}, ω_{ξ}, ω_a, ω^{α}, ω^{ξ}. It follows from here (and from the same identities) that the form ω^{ξ}_{a} is also principal, but it has expansion in terms of the principal forms ω_{α}, ω_{ξ}, ω_a, ω^{α} only. Comparison of these relations shows that the form ω^{ξ}_{a} has an expansion in terms of the principal forms ω^{α}, ω^{ξ} only. Therefore, in particular, $C^{\xi}_{ab} = 0$.

Further classification of admissible differential-geometric structures is based on the analysis of algebraic relations (2.9).

A) At least for one value of the index $i\,(= 1, \dots, n - m)$,

$$\det\left(C^{m+i}_{\xi\eta}\right) \neq 0 \neq \det\left(C^{\xi\eta}_{m+i}\right).$$

B) At least for one value of the index $i\,(= 1, \dots, n - m)$,

$$\det\left(C^{m+i}_{\xi\eta}\right) \neq 0 = \det\left(C^{\xi\eta}_{m+i}\right).$$

C) At least for one value of the index $i(=1,\dots,n-m)$,

$$\det\left(C^{m+i}_{\xi\eta}\right)=0=\det\left(C^{\xi\eta}_{m+i}\right).$$

The case (A) was studied in [13]. Let us study the case B. Taking into account that the matrix $\left(C^{m+i}_{\xi\eta}\right)$ is of maximal rank, it is easy to see from algebraic relations (2.9) that

$$C^{\xi}_{ab}=0,\quad C^{\xi}_{a\alpha}=0,\quad C^{\xi}_{a\eta}=0,\quad \text{therefore}\quad \omega^{\xi}_{a}=0.$$

It follows from identities (2.8) that the values $C^{m+i}_{\alpha\xi}$ can be considered equal to zero: $C^{m+i}_{\alpha\xi}=0$, then the forms ω^{ξ}_{α} become principal. Using the same procedure, we arrive at the relation $C^{\xi\alpha}_{m+i}=0$ but values $C^{\alpha\beta}_{\xi}$ remain arbitrary. This gives a reason to consider system of algebraic relations (2.9) once more and suppose that, for a fixed value of the index $x(=2m-n+1,\dots,n-m)$, the matrix $C^{\alpha\beta}_{\xi}$ is nondegenerate: $\det C^{\alpha\beta}_{\xi}\neq 0$. It follows from this condition that $C^{\xi\eta}_{m+i}=0$. Let us note that this condition can be obtained from the relation $\det\left(C^{m+i}_{\xi\eta}\right)\neq 0$ (for fixed value of the index i $(=1,\dots,n-m)$. It is easy to see now that the system of linear differential equations

$$\omega^{\xi}_{\alpha}=0,\quad \omega^{\xi}_{a}=0,\quad \omega^{\alpha}_{\beta}=0,\quad \omega^{\xi}_{\eta}=0,\quad \omega^{a}_{b}=0$$

is completely integrable. We can rewrite the system of structure equations of M in the following form

$$\begin{aligned}
&d\omega^{\alpha}=C^{\alpha\beta}_{\xi}\omega_{\beta}\wedge\omega^{\xi},\quad d\omega^{\xi}=0,\quad d\omega^{a}=\omega^{a}_{\alpha}\wedge\omega^{\alpha}+\omega^{a}_{\xi}\wedge\omega^{\xi},\\
&d\omega_{\alpha}=-\omega^{a}_{\alpha}\wedge\omega_{a},\quad d\omega_{\xi}=-\omega^{a}_{\xi}\wedge\omega_{a},\quad d\omega_{a}=0,\\
&d\omega^{a}_{\alpha}=C^{ab}_{m+i}C^{m+i}_{\alpha\beta}\omega_{b}\wedge\omega^{\beta},\quad d\omega^{a}_{\xi}=C^{\alpha\beta}_{\xi}\omega^{a}_{\alpha}\wedge\omega_{\beta}+C^{ab}_{m+i}C^{m+i}_{\xi\eta}\omega_{b}\wedge\omega^{\eta},
\end{aligned}\tag{2.3$'$}$$

where the coefficients satisfy equations (2.8). Exterior differentiation of the identity $\omega^{\xi}_{\alpha}=0$ gives the following algebraic condition

$$C^{m+i}_{\alpha\beta}C^{\alpha\beta}_{\xi}=0,$$

therefore, $C^{m+i}_{\alpha\beta}=0$. It is obvious now that the system of linear differential equations $\omega^{a}_{\alpha}=0$ is completely integrable. We obtain the final form of the system of structure equations

$$\begin{aligned}
&d\omega^{\alpha}=C^{\alpha\beta}_{\xi}\omega_{\beta}\wedge\omega^{\xi},\quad d\omega^{\xi}=0,\quad d\omega^{a}=\omega^{a}_{\xi}\wedge\omega^{\xi},\\
&d\omega_{\alpha}=0,\quad d\omega_{\xi}=-\omega^{a}_{\xi}\wedge\omega_{a},\quad d\omega_{a}=0,\\
&d\omega^{a}_{\xi}=C^{ab}_{m+i}C^{m+i}_{\xi\eta}\omega_{b}\wedge\omega^{\eta}.
\end{aligned}\tag{2.3$''$}$$

$$\begin{aligned}
&dC^{\alpha\beta}_{\xi}=C^{\alpha\beta\gamma}_{\xi}\omega_{\gamma},\\
&dC^{m+\alpha}_{\xi\eta}=C^{m+\alpha}_{\xi\eta\mu}\omega^{\mu},\\
&dC^{m+\mu}_{\xi\eta}=-C^{m+\alpha}_{\xi\eta}C^{\alpha\beta}_{\mu}\omega_{\beta}+C^{m+\alpha}_{\xi\eta\mu}\omega^{\mu},\\
&dC^{ab}_{m+\alpha}=C^{ab}_{m+\xi}C^{\alpha\beta}_{\xi}\omega_{\beta}+C^{abc}_{m+\alpha}\omega_{c},\\
&dC^{ab}_{m+\xi}=C^{abc}_{m+\xi}\omega_{c}.
\end{aligned}\tag{2.8$'$}$$

It is easy to see that the system of principal and secondary differential forms ω^{α}, ω^{ξ}, ω^{a}, ω_{α}, ω_{ξ}, ω_{a}, ω^{a}_{ξ} and functions $C^{\alpha\beta}_{\xi}$, $C^{m+i}_{\xi\eta}$, C^{ab}_{m+i} satisfying equations (2.3$''$), (2.8$'$) is closed and, therefore, by virtue of the Cartan – Laptev theorem [14] the following statement is true.

Theorem 2.1. *The metric connection of a* $2n$*-dimensional pseudo-Euclidean Rashevsky space* E^n_{2n} *induces a differential-geometric structure of special type affine connection determined by the system of differential forms* ω^α, ω^ξ, ω^a, ω_α, ω_ξ, ω_a, ω^a_ξ *and functions* $C^{\alpha\beta}_\xi$, $C^{m+i}_{\xi\eta}$, C^{ab}_{m+i}, $\alpha, \beta = 1, \dots, 2m-n$, $\xi, \eta = 2m-n+1, \dots, n-m$, $a, b = n-m+1, \dots, m$, $i = 1, \dots, n-m$ *satisfying equations* $(2.3'')$, $(2.8')$ *on* $2m$*-dimensional* $(15n < 30m = 18n)$ *submanifold* M *defined by equations* (2.1) *on condition that, at least for one value of the index* $i\,(=1, \dots, n-m)$, $\det\left(C^{m+i}_{\xi\eta}\right) \neq 0 = = \det\left(C^{\xi\eta}_{m+i}\right)$, $\det\left(C^{\alpha\beta}_\xi\right) \neq 0$.

The structure of this affine connection can be studied using structure equations $(2.3')$. Let us note that it has nontrivial curvature tensor

$$R^{ab}_{\xi\eta} = C^{ab}_{m+i} C^{m+i}_{\xi\eta}.$$

To study the structure on M, let us note that the system of linear differential equations $\omega^\xi = 0$ is completely integrable and, therefore, it determines submanifolds of dimension $n-m$. Therefore the following result is established.

Theorem 2.2. *The submanifold* M *is a double fiber bundle. The fibers of the first bundle are foliations with* $n-m$*-dimensional flat leaves. The fibers of the second bundle are cross products of* $2m-n$*- and* $n-m$*-dimensional planes in* E^n_{2n}.

It is easy to see that the system of Pfaff equations $\omega^\alpha = 0$, $\omega_\alpha = 0$, $\alpha = 1, \dots, 2m-n$ is completely integrable and determines in M submanifolds N of dimension $2(n-m)$.

3. Canonical integral

It is known [9] that a k-tuple integral depending on k parameters induces a structure of a pseudo-Riemannian Rashevsky space on the $2k$-dimensional manifold N of integration variables and parameters. The inverse problem of finding a k-tuple integral depending on k parameters inducing a given admissible structure on N, known as a canonical integral of this differential-geometric structure, is much more interesting. If the curvature tensor is trivial, this integral leads to the Fourier transform. It is evident that, in all other cases, obtained integrals are natural generalizations of the Fourier transform.

Let us find a canonical integral of a differential-geometric structure defined by equations $(2.3'')$, $(2.8')$ on N, i.e., an $n-m$-tuple integral of the form

$$\Omega = \lambda \omega^{2m-n+1} \wedge \cdots \wedge \omega^m \tag{3.1}$$

depending on $n-m$ parameters inducing the differential-geometric structure

$$\begin{aligned}
&d\omega^\xi = 0, \quad d\omega^a = \omega^a_\xi \wedge \omega^\xi,\\
&d\omega_\xi = -\omega^a_\xi \wedge \omega_a, \quad d\omega_a = 0,\\
&d\omega^a_\xi = C^{ab}_{m+i} C^{m+i}_{\xi\eta} \omega_b \wedge \omega^\eta,\\
&dC^{m+\alpha}_{\xi\eta} = C^{m+\alpha}_{\xi\eta\mu} \omega^\mu,\\
&dC^{m+\mu}_{\xi\eta} = C^{m+\alpha}_{\xi\eta\mu} \omega^\mu,\\
&dC^{ab}_{m+\alpha} = C^{abc}_{m+\xi} \omega_c,\\
&dC^{ab}_{m+\xi} = C^{abc}_{m+\xi} \omega_c.
\end{aligned} \tag{3.2}$$

on the $2(n-m)$-dimensional manifold of integration variables and parameters. Following the results obtained in [9], this procedure includes solution of the system of differential equations

$$\begin{aligned} &d\ln\lambda = \lambda_\xi\omega^\xi + \lambda_a\omega^a + \lambda^\xi\omega_\xi + \lambda^a\omega_a, \\ &d\left(\lambda^\xi\omega_\xi + \lambda^a\omega_a\right) = \omega^\xi\wedge\omega_\xi + \omega^a\wedge\omega_a + \delta^a_{2m-n+\xi}\omega_a\wedge\omega^\xi. \end{aligned} \tag{3.3}$$

Using structure equations $(2.3'')$, let us present the essential principal and secondary forms as linear combinations of differentials of variables:

$$\begin{aligned} &\omega^\xi = dx^\xi, \quad \omega^a = dx^a - \frac{1}{2}C^a_{m+i}C^{m+i}_\xi dx^\xi, \\ &\omega_\xi = dy_\xi - C^a_{m+i}C^{m+i}_\xi dy_a, \quad \omega_a = dy_a, \\ &\omega^a_\xi = \frac{1}{2}\left(C^a_{m+i}C^{m+i}_{\xi\eta}dx^\eta - C^{ab}_{m+i}C^{m+i}_\xi dy_b\right) \end{aligned} \tag{3.4}$$

where the smooth functions $C^a_{m+i} = C^a_{m+i}(y_{2m-n+1},\ldots,y_m)$, $C^{m+i}_\xi = C^{m+i}_\xi(x^{2m-n+1},\ldots,x^m)$ are solutions of the following differential equations

$$\begin{aligned} dC^a_{m+i} &= C^{ab}_{m+i}dy_b, \\ dC^{m+i}_\xi &= C^{m+i}_{\xi\eta}dx^\eta. \end{aligned}$$

It is easy to check that forms (3.4) satisfy structure equations $(2.3'')$.

Let us introduce the following formal expansions of the differentials of the coefficients of equations (3.3):

$$\begin{aligned} d\lambda^\xi &= \lambda^\xi_\eta\omega^\eta + \lambda^\xi_a\omega^a + \lambda^{\xi\eta}\omega_\eta + \lambda^{\xi a}\omega_a, \\ d\lambda^a &= \lambda^a_\xi\omega^\xi + \lambda^a_b\omega^b + \lambda^{a\xi}\omega_\xi + \lambda^{ab}\omega_b, \\ d\lambda_\xi &= \mu_{\xi\eta}\omega^\eta + \mu_{\xi a}\omega^a + \mu^\eta_\xi\omega_\xi + \mu^a_\xi\omega_a, \\ d\lambda_a &= \mu_{a\xi}\omega^\xi + \mu_{ab}\omega^b + \mu^\xi_a\omega_\xi + \mu^b_a\omega_b. \end{aligned} \tag{3.5}$$

Let us substitute now the expressions of basic forms into these expansions. Then we substitute these expansions into the second relation and into the result of exterior differentiation of the first relation of system (3.3). As a result we obtain the following system of algebraic relations:

$$\begin{aligned} &\mu_{\xi\eta} = \mu_{\eta\xi}, \quad \mu_{\xi a} = \mu_{a\xi}, \quad \mu_{ab} = \mu_{ba}, \quad \lambda^{\xi\eta} = \lambda^{\eta\xi}, \quad \lambda^{\xi a} = \lambda^{a\xi}, \\ &\lambda^\xi_\eta = \mu^\xi_\eta = \delta^\xi_\eta, \quad \lambda^a_b = \mu^a_b = \delta^a_b, \quad \mu^\xi_a = \lambda^\xi_a = 0, \\ &\mu^a_\xi = \frac{1}{2}C^{ab}_{m+i}C^{m+i}_\xi\lambda_b - \delta^a_{2m-n+\xi}, \quad \lambda^a_\xi = \frac{1}{2}C^a_{m+i}C^{m+i}_{\xi\eta}\lambda^\eta - \delta^a_{2m-n+\xi}. \end{aligned}$$

Substitution of the obtained relations into the system of expansions (3.5) gives the following system of differential equations

$$\begin{aligned} &\frac{\partial\lambda^\xi}{\partial x^\eta} = \delta^\xi_\eta, \quad \frac{\partial\lambda^\xi}{\partial x^a} = 0, \quad \frac{\partial\lambda^\xi}{\partial y_\eta} = \lambda^{\xi\eta}, \quad \frac{\partial\lambda^\xi}{\partial y_a} = -\lambda^{\xi\eta}C^a_{m+i}C^{m+i}_\eta; \\ &\frac{\partial\lambda^a}{\partial x^\xi} = -\delta^a_{2m-n+\xi} + \frac{1}{2}C^a_{m+i}C^{m+i}_{\xi\eta}\lambda^\eta - \frac{1}{2}C^a_{m+i}C^{m+i}_\xi; \\ &\frac{\partial\lambda^a}{\partial x^b} = \delta^a_b, \quad \frac{\partial\lambda^a}{\partial y_\xi} = \lambda^{\xi a}, \quad \frac{\partial\lambda^a}{\partial y_b} = \lambda^{ab} - \lambda^{\xi a}C^b_{m+i}C^{m+i}_\xi; \end{aligned}$$

$$\frac{\partial \lambda_\xi}{\partial x^\eta} = \mu_{\xi\eta} - \frac{1}{2}\mu_{\xi a} C^a_{m+i} C^{m+i}_\eta, \quad \frac{\partial \lambda_\xi}{\partial x^a} = \mu_{\xi a}, \quad \frac{\partial \lambda_\xi}{\partial y_\eta} = \delta^\eta_\xi,$$

$$\frac{\partial \lambda_\xi}{\partial y_a} = \frac{1}{2} C^{ab}_{m+i} C^{m+i}_\xi \lambda_b - \frac{1}{2} C^a_{m+i} C^{m+i}_\xi - \delta^a_{2m-n+\xi};$$

$$\frac{\partial \lambda_a}{\partial x^\xi} = \mu_{\xi a} - \frac{1}{2}\mu_{ab} C^a_{m+i} C^{m+i}_\xi, \quad \frac{\partial \lambda_a}{\partial x^b} = \mu_{ab}, \quad \frac{\partial \lambda_a}{\partial y_\xi} = 0, \quad \frac{\partial \lambda_a}{\partial y_b} = \delta^b_a.$$

Solving this system, we represent the solution in the following form

$$\begin{aligned}
\lambda^\xi &= x^\xi + \psi^\xi(y), \\
\lambda^a &= x^a - \delta^a_{2m-n+\alpha} - \frac{1}{2} C^a_{m+i} C^{m+i}_\xi x^\xi + \psi^a(y), \\
\lambda_\xi &= y_\xi - \frac{1}{2} C^a_{m+i} C^{m+i}_\xi y_a - \delta^a_{2m-n+\xi} y_a + \varphi_\xi(x), \\
\lambda_a &= y_a + \varphi_a(x),
\end{aligned}$$

where $\varphi_\xi(x)$, $\varphi_a(x)$, $\psi^\xi(y)$, $\psi^a(y)$, are smooth functions of the corresponding variables. Substitution of these expressions into the first equation of system (3.3) gives the formula

$$\ln \lambda = x^\xi y_\xi + x^a y_a - \delta^a_{2m-n+\xi} x^\xi y_a - C^a_{m+i} C^{m+i}_\xi x^\xi y_a + \varphi(x) + \psi(y),$$

where $\varphi(x) = \varphi(x^1, \ldots, x^m)$ and $\psi(y) = \psi(y_1, \ldots, y_m)$ are smooth functions on the corresponding fibers of the double bundle N. Therefore, the following result holds.

Theorem 3.1. *An $n-m$-tuple integral depending on $n-m$ parameters inducing a differential-geometric structure (3.2) on the $2(n-m)$-dimensional submanifold N of variables and parameters can be reduced to an integral of the form*

$$\Omega = P(x)Q(y) \exp\Big[x^\xi y_\xi + x^a y_a - \delta^a_{2m-n+\xi} x^\xi y_a - C^a_{m+i} C^{m+i}_\xi x^\xi y_a\Big] \times \\ \times dx^{2m-n+1} \wedge \cdots \wedge dx^m \quad (3.6)$$

where $P(x) = P(x^{2m-n+1}, \ldots, x^m)$ and $Q(y) = Q(y_{2m-n+1}, \ldots, y_m)$ are the exponents of functions $\varphi(x) = \varphi(x^{2m-n+1}, \ldots, x^m)$ and $\psi(y) = \psi(y_{2m-n+1}, \ldots, y_m)$ respectively.

In the special case when the values $C^{m+i}_{\xi\eta}$, C^{ab}_{m+i} are constants, we arrive at the formulas

$$C^a_{m+i} = C^{ab}_{m+i} y_b, \quad C^{m+i}_\xi = C^{m+i}_{\xi\eta} x^\eta,$$

and expression (3.6) can be rewritten in the following more symmetric form

$$\Omega = P(x)Q(y) \exp\Big[x^\xi y_\xi + x^a y_a - \delta^a_{2m-n+\xi} x^\xi y_a - C^{ab}_{m+i} C^{m+i}_{\xi\eta} x^\xi x^\eta y_a y_b\Big] \times \\ \times dx^{2m-n+1} \wedge \cdots \wedge dx^m. \quad (3.7)$$

Let us note that the partial derivatives of the functions C^a_{m+i}, C^{m+i}_ξ occurring in the canonical integral of a differential-geometric structure (3.6) compose the curvature tensor of the corresponding affine connection. Besides, the components of the curvature tensor are coefficients of monoms of the fourth degree.

4. Parametric equations

To find parametric equations of the submanifold M under consideration, let us integrate structure equations $(2.3'')$. To do this, let us consider the equations of infinitesimal displacement of a moving frame $(P, e^\alpha, e^\xi, e^a, e^{m+i}, e_\alpha, e_\xi, e_a, e_{m+i})$ in the space of affine connection E^n_{2n}

$$\begin{aligned}
de^\alpha &= \omega^\alpha_\beta e^\beta + \omega^\alpha_\xi e^\xi + \omega^\alpha_a e^a + \omega^\alpha_{m+i} e^{m+i},\\
de^\xi &= \omega^\xi_\beta e^\beta + \omega^\xi_\eta e^\xi + \omega^\xi_a e^a + \omega^\xi_{m+i} e^{m+i},\\
de^a &= \omega^a_\alpha e^\alpha + \omega^a_\xi e^\xi + \omega^a_b e^b + \omega^a_{m+i} e^{m+i},\\
de^{m+i} &= \omega^{m+i}_\alpha e^\alpha + \omega^{m+i}_\xi e^\xi + \omega^{m+i}_a e^a + \omega^{m+i}_{m+k} e^{m+k},\\
de_\xi &= -\omega^\alpha_\xi e_\alpha - \omega^\eta_\xi e_\eta - \omega^a_\xi e_a - \omega^{m+k}_\xi e_{m+k},\\
de_a &= -\omega^\alpha_a e_\alpha - \omega^\xi_a e_\xi - \omega^b_a e_b - \omega^{m+k}_a e_{m+k},\\
de_{m+i} &= -\omega^\alpha_{m+i} e_\alpha - \omega^\xi_{m+i} e_\xi - \omega^a_{m+i} e_a - \omega^{m+k}_{m+i} e_{m+k}.
\end{aligned}$$

The submanifold M is characterized by the identities

$$\omega^\alpha_\beta = 0, \quad \omega^a_b = 0, \quad \omega^\xi_\eta = 0, \quad \omega^\alpha_a = 0, \quad \omega^\alpha_{m+i} = 0, \quad \omega^\xi_\alpha = 0,$$

$$\omega^\xi_a = 0, \quad \omega^\xi_{m+i} = 0, \quad \omega^a_\alpha = 0, \quad \omega^{m+i}_\alpha = 0, \quad \omega^{m+i}_a = 0,$$

$$\alpha, \beta = 1, 2m-n; \quad \xi, \eta = 2m-n+1, \dots, n-m; \quad a, b = n-m+1, \dots, m.$$

Substitution of these relations into the previous system gives the following equations of infinitesimal displacement of a moving frame $(P, e^\alpha, e^\xi, e^a, e^{m+i}, e_\alpha, e_\xi, e_a, e_{m+i})$ on the space of affine connection M:

$$\begin{aligned}
de^\alpha &= C^{\alpha\beta}_\xi dy_\beta e^\xi,\\
de^\xi &= 0,\\
de^a &= \frac{1}{2}\left(C^a_{m+i} C^{m+i}_{\xi\eta} dx^\eta - C^{ab}_{m+i} C^{m+i}_\xi dy_b\right) e^\xi + C^{ab}_{m+i} dy_b e^{m+i},\\
de^{m+i} &= C^{m+i}_{\alpha\beta} dx^\beta e^\alpha + C^{m+i}_{\xi\eta} dx^\eta e^\xi + \omega^{m+i}_{m+k} e^{m+k},\\
de_\alpha &= 0,\\
de_\xi &= -C^{\alpha\beta}_\xi dy_\beta e_a - \frac{1}{2}\left(C^a_{m+i} C^{m+i}_{\xi\eta} dx^\eta - C^{ab}_{m+i} C^{m+i}_\xi dy_b\right) e_a - C^{m+i}_{\xi\eta} dx^\eta e_{m+i},\\
de_a &= 0,\\
de_{m+i} &= -C^{ab}_{m+i} dy_b e_a - \omega^{m+k}_{m+i} e_{m+k}.
\end{aligned}$$

Replacing the secondary forms ω^{m+i}_{m+k} by $-\omega^k_i$ and solving the obtained, we obtain the following expressions for the basis vectors

$$\begin{aligned}
e^\alpha &= C^\alpha_\xi (e^\xi)_0 + (e^\alpha)_0,\\
e^\xi &= (e^\xi)_0,
\end{aligned}$$

$$e^a = \left[C^a_{m+i}C^{m+i}_\beta - C^a_{m+\xi}C^\alpha_\xi C^{m+\alpha}_\beta\right](e^\beta)_0 +$$
$$+\left[\frac{1}{2}C^a_{m+i}C^{m+i}_\xi + C^a_{m+i}\left(C^{m+i}_\beta C^\beta_\xi + C^{m+i}_\xi\right)\right](e^\xi)_0 +$$
$$+\left(C^a_{m+\alpha} - C^a_{m+\xi}C^\alpha_\xi\right)(e^{m+\alpha})_0 + C^a_{m+\xi}(e^{m+\xi})_0 + (e^a)_0,$$
$$e^{m+\alpha} = C^{m+\alpha}_\beta(e^\beta)_0 + \left(C^{m+\alpha}_\beta C^\beta_\xi + C^{m+\alpha}_\xi\right)(e^\xi)_0 + (e^{m+\alpha})_0,$$
$$e^{m+\xi} = \left(C^{m+\xi}_\beta - C^\alpha_\xi C^{m+\alpha}_\beta\right)(e^\beta)_0 + \left(C^{m+\xi}_\alpha C^\alpha_\eta + C^{m+\xi}_\eta\right)(e^\eta)_0 - C^\alpha_\xi(e^{m+\alpha})_0 + (e^{m+\xi})_0,$$
$$e_\alpha = (e_\alpha)_0,$$
$$e_\xi = -C^\alpha_\xi(e_\alpha)_0 + \left(\frac{1}{2}C^{m+i}_\xi C^a_{m+i} + C^{m+\alpha}_\xi C^\alpha_\eta C^a_{m+\eta}\right)(e_a)_0 - C^{m+\alpha}_\xi(e_{m+\alpha})_0 +$$
$$+\left(C^{m+\eta}_\xi + C^{m+\alpha}_\xi C^\alpha_\eta\right)(e_{m+\eta})_0 + (e_\eta)_0,$$
$$e_a = (e_a)_0,$$
$$e_{m+\alpha} = -\left(C^a_{m+\alpha}C^\alpha_\xi C^a_{m+\xi}\right)(e_a)_0 + C^\alpha_\xi(e_{m+\xi})_0 + (e_{m+\alpha})_0,$$
$$e_{m+\xi} = -C^a_{m+\xi}(e_a)_0 + (e_{m+\xi})_0.$$

where $\left[P,(e^\alpha)_0,(e^\xi)_0,(e^a)_0,(e^{m+\alpha})_0,(e^{m+\xi})_0,(e_\alpha)_0,(e_\xi)_0,(e_a)_0,(e_{m+\alpha})_0,(e_{m+\xi})_0\right]$ is a fixed orthonormal frame in E^n_{2n} . Substitution of these relations into the equation

$$dP = -\omega^\alpha e_\alpha - \omega^\xi e_\xi - \omega^a e_a - \omega_{2m-n+i}e_{m+i} - \omega_\alpha e^\alpha - \omega_\xi e^\xi - \omega_a e^a - \omega^i e^{m+i}$$

and further integration gives the equality

$$P = -x^\alpha(e^\alpha)_0 - x^\xi(e^\xi)_0 -$$
$$-\Big[x^a - C^{m+\alpha}_\xi C_{m+\eta}C^\alpha_\eta - \delta^\xi_{2m-n+\alpha}\left(C^a_{m+i} + C^\alpha_\eta C^a_{m+\eta}\right)y_\eta +$$
$$+\delta^\xi_{2m-n+\alpha}\left(C^a_{m+\alpha} + C^\alpha_\eta C^a_{m+\eta}\right)C_{m+i}C^{m+i}_\xi -$$
$$-\delta^b_{2m-n+\alpha}\left(C^a_{m+\alpha} + C^\alpha_\xi C^a_{m+\xi}\right)y_b - \delta^b_{2m-n+\xi}C^a_{m+\xi}y_b\Big](e_a)_0 -$$
$$-\left[\delta^\xi_{2m-n+\alpha}y_\xi + C_{m+\alpha} + \delta^a_{2m-n+\alpha}y_a - \delta^\xi_{2m-n+\alpha}C_{m+i}C^{m+i}_\xi\right](e_{m+\alpha})_0 -$$
$$-\Big[\delta^a_{2m-n+\xi}y_a + \delta^a_{2m-n+\alpha}C^\alpha_\xi y_a - \delta^\eta_{2m-n+\alpha}C_{m+i}C^{m+i}_\eta C^\alpha_\xi + \delta^\eta_{2m-n+\alpha}C^\alpha_\xi y_\eta +$$
$$+\left(C^{m+\xi} + C^{m+\alpha}C^\alpha_\xi\right)\Big](e_{m+\xi})_0 - \Big[y_\alpha + \left(C^{m+\xi} - C^\beta_\xi C^{m+\beta}_\alpha\right)x^\xi +$$
$$+\left(C_{m+i}C^{m+i}_\alpha - C_{m+\xi}C^\beta_\xi C^{m+\beta}_\alpha\right) + C_\alpha + C^\beta_\xi C^{m+\beta}_\alpha x^\xi\Big](e^\alpha)_0 -$$
$$-\Big[y_\xi + C_\xi + \left(-\frac{1}{2}C_{m+i}C^{m+i}_\xi + C_{m+i}\left(C^{m+i}_\beta C^\beta_\xi + C^{m+i}_\xi\right)\right) + \left(C_\beta C^\beta_\xi + C_\xi\right) +$$
$$+C^\alpha_\eta\left(C^{m+\alpha}_\beta C^\beta_\xi x^\eta + C^{m+\alpha}_\eta\right) + (\cdots + C_\xi)\Big](e^\xi)_0 - y_a(e^a)_0 -$$
$$-\left(x^\alpha + C_{m+\alpha} - C_{m+\xi}C^\alpha_\xi\right)(e^{m+\alpha})_0 - \left(x^\xi + C_{m+\xi}\right)(e^{m+\xi})_0.$$

Therefore, the following statement is true.

Theorem 4.1. *The submanifold M has be given by parametric equations of the form*

$$X^\alpha = x^\alpha, \quad X^\xi = x^\xi,$$

$$X^a = x^a - C_\xi^{m+\alpha} C_{m+\eta} C_\eta^\alpha - \delta_{2m-n+\alpha}^\xi \left(C_{m+i}^a + C_\eta^\alpha C_{m+\eta}^a\right) y_\eta +$$
$$+ \delta_{2m-n+\alpha}^\xi \left(C_{m+\alpha}^a + C_\eta^\alpha C_{m+\eta}^a\right) C_{m+i} C_\xi^{m+i} -$$
$$- \delta_{2m-n+\alpha}^b \left(C_{m+\alpha}^a + C_\xi^\alpha C_{m+\xi}^a\right) y_b - \delta_{2m-n+\xi}^b C_{m+\xi}^a y_b,$$

$$X^{m+\alpha} = \delta_{2m-n+\alpha}^\xi y_\xi + C_{m+\alpha} + \delta_{2m-n+\alpha}^a y_a - \delta_{2m-n+\alpha}^\xi C_{m+i} C_\xi^{m+i},$$

$$X^{m+\xi} = \delta_{2m-n+\xi}^a y_a + \delta_{2m-n+\alpha}^a C_\xi^\alpha y_a - \delta_{2m-n+\alpha}^\eta C_{m+i} C_\eta^{m+i} C_\xi^\alpha +$$
$$+ \delta_{2m-n+\alpha}^\eta C_\xi^\alpha y_\eta + C^{m+\xi} + C^{m+\alpha} C_\xi^\alpha,$$

$$Y_a = y_\alpha + \left(C^{m+\xi} - C_\xi^\beta C_\alpha^{m+\beta}\right) x^\xi + \left(C_{m+i} C_\alpha^{m+i} - C_{m+\xi} C_\xi^\beta C_\alpha^{m+\beta}\right) +$$
$$+ C_\alpha + C_\xi^\beta C_\alpha^{m+\beta} x^\xi,$$

$$Y_\xi = y_\xi + C_\xi + \left[\frac{1}{2} C_{m+i} C_\xi^{m+i} + C_{m+i}\left(C_\beta^{m+i} C_\xi^\beta + C_\xi^{m+i}\right)\right] + C_\beta C_\xi^\beta +$$
$$+ C_\eta^\alpha \left(C_\beta^{m+\alpha} C_\xi^\beta x^\eta + C_\eta^{m+\alpha}\right) + C_\alpha^{m+\eta} C_\xi^\alpha,$$

$$Y_a = y_a, \quad Y_{m+\alpha} = x^\alpha + C_{m+\alpha} - C_{m+\xi} C_\xi^\alpha, \quad Y_{m+\xi} =^\xi + C_{m+\xi},$$

where

$$C^{m+\alpha} = C^{m+\alpha}(x^1, \ldots, x^{2m-n}), \quad C^{m+\xi} = C^{m+\xi}(x^1, \ldots, x^{2m-n}),$$
$$C_\alpha = C_\alpha(x^1, \ldots, x^{2m-n}), \quad C_\xi = C_\xi(y_{n-m+1}, \ldots, y_m),$$
$$C_{m+\alpha} = C_{m+\alpha}(y_{n-m+1}, \ldots, y_m), \quad C_{m+\xi} = C_{m+\xi}(y_{n-m+1}, \ldots, y_m)$$

are smooth functions satisfying the following differential equations

$$dC^{m+\alpha} = C_\beta^{m+\alpha} dx^\beta, \quad dC^{m+\xi} = C_\alpha^{m+\xi} dx^\alpha, \quad dC_\alpha = C_\alpha^{m+\beta} dx^\beta,$$
$$dC_{m+\alpha} = C_{m+\alpha}^a dy_a, \quad dC_{m+\xi} = C_{m+\xi}^a dy_a.$$

It is easy to check that the parametric equations of the submanifolds $N \subset M$ can be written in the following form

$$X^\xi = x^\xi,$$

$$X^a = x^a - \delta_{2m-n+\alpha}^\xi C_{m+i}^a y_\eta + \delta_{2m-n+\alpha}^\xi C_{m+\alpha}^a C_{m+i} C_\xi^{m+i} - \delta_{2m-n+\alpha}^b C_{m+\xi}^a y_b,$$

$$X^{m+\alpha} = \delta_{2m-n+\alpha}^\xi y_\xi + C_{m+\alpha} + \delta_{2m-n+\alpha}^a y_a - \delta_{2m-n+\alpha}^\xi C_{m+i} C_\xi^{m+i},$$

$$X^{m+\xi} = \delta_{2m-n+\xi}^a y_a + C^{m+\xi}, \quad Y_\xi = y_\xi + \frac{1}{2} C_{m+i} C_\xi^{m+i} + C_{m+i} C_\xi^{m+i}, \quad Y_a = y_a,$$

$$Y_{m+\alpha} = x^\alpha + C_{m+\alpha}, \quad Y_{m+\xi} = x^\xi + C_{m+\xi},$$

where $C_{m+\alpha} = C_{m+\alpha}(y_{n-m+1}, \ldots, y_m)$, $C_{m+\xi} = C_{m+\xi}(y_{n-m+1}, \ldots, y_m)$ are smooth functions satisfying the following differential equations

$$dC_{m+\alpha} = C_{m+\alpha}^a dy_a, \quad dC_{m+\xi} = C_{m+\xi}^a dy_a.$$

Резюме

С.Х. Арутюнян. О геометрии подмногообразий в E^n_{2n}.

Изучается специальный класс $2m$-мерных подмногообразий в $2n$-мерном псевдоевклидовом пространстве с метрикой сигнатуры (n,n), известном как псевдоевклидово пространство Рашевского. Для изучаемых подмногообразий найдены канонические интегралы и параметрические уравнения.

Ключевые слова: четномерное подмногообразие, псевдоевклидово пространство Рашевского, двойное расслоение, канонический интеграл, дифференциально-геометрическая структура, расслоение, слоение.

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