Controlling the frequencies of photons emitted by a single quantum dot in a one-dimensional photonic crystal

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Abstract

Subject of study. **A single InAs quantum dot in a one-dimensional photonic crystal based on GaAs is examined.** ***Aim of study.*** **The aim of this study is to develop a method for controlling photon emission frequencies from a single quantum dot within a one-dimensional photonic crystal based on changes in the electromagnetic mass of an electron in the photonic crystal medium.** ***Method.*** **The proposed approach leverages the effect of changing the electromagnetic mass of an electron in the photonic crystal medium, manifesting as corrections to electron energy levels depending on the optical density of the medium. To control this density, the injection of free charge carriers and the quadratic electro-optic Kerr effect are proposed.** ***Main results.*** **The feasibility of in situ control of photon emission frequencies from a quantum dot was demonstrated using quantum transitions between the *p*- and *s*-states of a hydrogen-like InAs quantum dot situated in the air voids of a one-dimensional GaAs photonic crystal. This control is achieved through the effect of changing the electromagnetic mass of an electron, as well as tuning the refractive index of the photonic crystal via free charge carrier injection and the electro-optic Kerr effect. Calculations indicate that the photon energy control range available in experiments is limited to several tens of microelectronvolts, restricting practical applicability, and the observed displacement effect is smaller than experimentally recorded values. However, the energy level displacement, influenced by the quantum electrodynamic effect under investigation, exhibits a quadratic dependence on the refractive index of the material forming the photonic crystal. Consequently, the method is expected to scale significantly with increasing optical density. Such photonic crystals could be constructed using metamaterials with a high refractive index.** ***Practical significance.*** **The findings of this study, centered on developing a method for controlling photon emission frequencies from a single quantum dot in a one-dimensional photonic crystal, lay the groundwork for photon-emitter interfaces. These interfaces will**

incorporate key quantum functionalities, including photonic qubits, single-photon light sources, and nonlinear quantum photon-photon gates.

1. INTRODUCTION

Quantum dots (QDs) are a promising platform for creating essential components of quantum computers [1–3], such as efficient single-photon sources [4–8], detectors [9], nanophotonic and nanoplasmonic devices [10], photovoltaics [11], and biological applications [12]. These applications rely on the unique optical properties of QDs, including a broad absorption spectrum, narrow spectral emission line, tunable emission wavelength due to quantum size effects, high photostability, and a high fluorescence quantum yield [10].

In the past decade, QDs embedded in the periodic structure of photonic crystals (PCs) have garnered significant attention [13]. PCs, composed of periodic arrays of optical resonators, exhibit sharp peaks in the photon density of states and spatial redistribution of the electromagnetic field, leading to a strong coupling effect between quantum emitters and the electromagnetic field. These systems are promising for creating photon qubits, microlasers [14], single-photon sources with tunable photon frequencies [15], and for observing and examining fundamental quantum electrodynamics (QED) effects [3, 16]. The advantage of QD-PC systems lies in their extremely small optical mode volume and seamless integration with optical waveguides and on-chip electronics [13, 17]. However, modifying the emission wavelength of QDs typically requires fabricating new dots with different parameters. Therefore, developing methods to control QD spectra in situ is an urgent challenge. This paper aims to present a method for controlling photon emission frequencies from a single quantum dot in a one-dimensional photonic crystal, based on altering the electromagnetic mass of the electron within the photonic crystal medium.

This study investigates the control of the emission spectrum of a single InAs QD embedded in a one-dimensional GaAs PC [17]. The proposed method is based on altering the electromagnetic mass of the electron within the PC medium [18]. This alteration results in corrections to the electron energy levels, which are influenced by the optical density of the medium. To control this density, we propose using charge carrier injection and the quadratic electro-optic Kerr effect [19].

It is well established [20] that the interaction between an electron in a vacuum cavity of a PC and its own radiation field is sensitive to changes in the electromagnetic modes of the structure. However, this interaction is typically confined to the Lamb shift [21]. In [18], it was demonstrated that this modification also affects the formation mechanism of the electromagnetic mass of a free electron. This results in a correction δm_{pc} to the electron's self-energy, which cannot be masked by the physical mass of the charged particle. In the context of atoms, this effect leads to shifts in energy levels and significant changes in ionization energy [22]. Notably, the QED effect being examined is amplified when the PC medium is composed of metamaterials with a high refractive index [23, 24]. This is because the correction δm_{pc} to the electromagnetic mass depends quadratically on the refractive index of

the optically dense components of the PC. This correction, which is anisotropic relative to the electron's momentum direction, depends on the state of the charged particle and is several orders of magnitude larger than the Lamb shift in a vacuum [22].

The aim of this study is to develop a method for controlling photon emission frequencies from a single quantum dot in a one-dimensional photonic crystal based on altering the electromagnetic mass of the electron within the photonic crystal medium.

2. MODIFICATION OF THE INTERACTION BETWEEN THE ELECTRON AND ITS OWN RADIATION FIELD IN A ONE-DIMENSIONAL PC

For a long time, the electromagnetic mass of the electron remained a mystery due to its divergent nature, making it impossible to measure experimentally [25]. Later, the renormalization principle was introduced as part of solving the Lamb shift problem [26]. According to this principle, the physical mass of the electron, m_e, is expressed as the sum of the bare mass m_0 and the electromagnetic mass m_{em}:

add rp70. (1)

In [18], it was demonstrated that modifying the interaction of an electron in the vacuum cavities of a PC with its own radiation field results in a change in the electromagnetic mass of the electron. In the effect under investigation, the electromagnetic mass of the electron explicitly manifests for the first time.

Since this new correction is observable, it must correspond to a specific quantum mechanical operator. In [18], an expression for this operator was derived for the case of a three-dimensional PC:

add rp70, (2)

where α denotes the fine-structure constant; $\hat{\mathbf{I}}_p = \hat{\mathbf{p}}/|\hat{\mathbf{p}}|$ denotes the operator of the momentum direction of the electron; the eigenvectors $\mathbf{E}_{kn}(\mathbf{G})$ are the amplitudes of Bloch plane waves $\mathbf{E}_{kn}(\mathbf{r}) = \Sigma_{\mathbf{G}}\mathbf{E}_{kn}(\mathbf{G})e^{i(\mathbf{k}+\mathbf{G})\mathbf{r}}$, and their corresponding eigenvalues $\omega_{kn}(\mathbf{k})$ are the dispersion relations [27]. Here, n denotes a band (state) number, and $\boldsymbol{\varepsilon}_\lambda(\mathbf{k})$ denotes the unit vector of the field polarization (λ) in a vacuum. The wave vector $\mathbf{k}$ is limited by the first Brillouin zone (FBZ), and $\mathbf{G}$ denotes the reciprocal lattice vector of the PC ($\mathbf{G} = N_1\mathbf{b}_1 + N_2\mathbf{b}_2 + N_3\mathbf{b}_3$), where $\mathbf{b}_i$ denotes the primitive basis vectors of the reciprocal lattice. The first term on the right-hand side of Eq. (2) represents the electromagnetic mass of the electron in the PC medium, while the second term represents the electromagnetic mass in a vacuum.

In this study, we focus on a one-dimensional photonic crystal (PC). Such media are simpler and more convenient from both experimental and theoretical perspectives. For example, the polarization structure of the electromagnetic field is explicitly defined from the outset:

add rp70, (3)

where $\varepsilon_1(\mathbf{k}_G)$ and $\varepsilon_2(\mathbf{k}_G)$ denote the unit vectors of transverse electric (TE) and transverse magnetic (TM) polarization, respectively, and $\mathbf{k}_G = \mathbf{k} + G\mathbf{e}_z$.

The operator for the self-energy correction to the electromagnetic mass of a free electron placed in the vacuum cavities of a one-dimensional PC with cylindrical symmetry was derived in [22] and is given by:

add rp70, (4)

where $\hat{\mathbf{I}}_{pc}$ denotes the unit vector directed perpendicular to the layers of the one-dimensional PC,

add rp70.

Here, $\omega_{\mathbf{kn1}}$ and $\omega_{\mathbf{kn2}}$ represent the dispersion relations for Bloch modes with TE and TM polarization, respectively. The eigenstates of the operator [Eq. (4)] are states with a defined momentum. For an atomic electron in the state $|\Psi\rangle$, we apply first-order perturbation theory, where the corresponding correction is the matrix element $\langle\Psi|\delta m_{\mathrm{pc}}(\hat{\mathbf{I}}_{\mathbf{p}})|\Psi\rangle$.

3. HAMILTONIAN OF ATOMS IN PHOTONIC CRYSTALS AND THE PHOTONIC CORRECTION TO THE ELECTROMAGNETIC MASS OF AN ELECTRON IN *s*- AND *p*-STATES

Thus, the Hamiltonian of an atom in a PC medium must be augmented with operators δm_{pc} $\mathbf{(\hat{I}_p)}$ for each electron. The effect on the nucleus must also be considered, although it is often negligible in atomic physics processes. In the case of a hydrogen atom, the Hamiltonian assumes its simplest form:

add rp71, (5)

where $\hat{H}$ denotes the Hamiltonian of a hydrogen atom in free space (in a vacuum) with eigenvalues E_{i} and eigenvectors $|\Psi_{\mathrm{i}}\rangle$. The atomic states and energies are defined by the Schrödinger equation, as follows:

add rp71. (6)

In first-order perturbation theory, we will assume that the eigenvectors of the atomic system in the PC medium remain unchanged, $|\Psi_{\mathrm{i,pc}}^{(1)}\rangle = |\Psi_i\rangle$. Thus, the expression for the energy of the atom can be written as follows:

add rp71. (7)

It should be noted that the correction $E_{\mathrm{i}}^{(1)} - E_{\mathrm{i}}$ depends only on the orbital quantum number l and its magnetic projection m_{l} quantum numbers:

$\langle\Psi|\delta m_{\mathrm{pc}}\,(\hat{\mathbf{I}}_{\mathbf{p}})|\,\Psi\rangle = \langle l, m_{\mathrm{l}}\,|\delta m_{\mathrm{pc}}\,(\hat{\mathbf{I}}_{\mathbf{p}})|\,l, m_{\mathrm{l}}\rangle$ [26].

In considering QDs, a one-electron approximation is employed, wherein the multi-electron problem is reduced to a one-electron problem, and the influence of all other electrons on the electron under consideration is described by a self-consistent periodic field [28]. In this study, this approximation is applied to a single InAs QD [29] with a hydrogen-like energy level structure. The transitions between the *p*-state and *s*-state, which form the conduction and valence bands in InAs, respectively, are investigated.

The wavefunctions of the electron in momentum representation $\Psi_{l,ml}(\mathbf{p})$ for the *s*-state ($l = 0$, $m_l = 0$) and *p*-states [$p_0(l = 1, m_l = 0)$ and $p_{\pm1}(l = 1, m_l = \pm1)$] are expressed as follows:

add rp71, (8)

add rp71, (9)

add rp71, (10)

where *p* denotes the absolute value of the electron momentum, and Θ and Φ denote the zenith and azimuthal angles, respectively [28]. Using Eq. (4), the matrix elements of the operator of the photonic crystal correction to the electromagnetic mass of the electron for its various states [Eqs. (8)–(10)] were calculated.

add rp71, (11)

add rp71, (12)

add rp71. (13)

Equations (11)–(13), as presented above, still contain ultraviolet divergences. This issue arises because the fundamental theory used to calculate the mode composition of the photonic crystal (PC) focuses solely on the optical frequency range, often neglecting material dispersion [27]. However, at higher frequencies, all materials exhibit transparency, which ensures the convergence of the integrals mentioned above. Convergence can be achieved either by introducing a cutoff parameter specifically selected for a particular medium [18] or by segmenting the frequency range into decreasing intervals with a constant refractive index [22].

The energy shift of the spectral line emitted by an atom or QD is determined by the difference in the corrections provided above:

add rp72, (14)

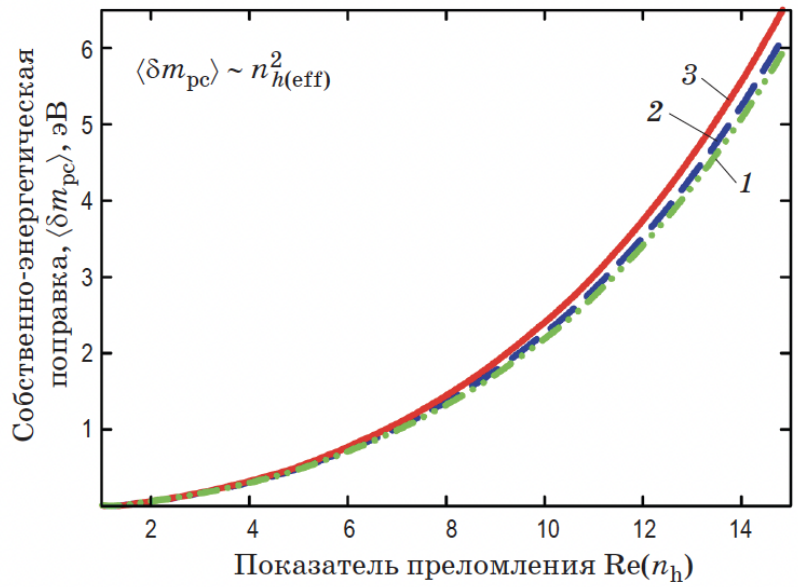
add rp72. (15)

4. CONTROL OF THE EMISSION SPECTRUM OF A SINGLE QD IN A ONE-DIMENSIONAL PC

The quadratic dependence on the refractive index of the PC layers (Fig. 1) is a significant characteristic of the self-energy correction δm_{pc}. This sensitivity allows for controlling the spectral lines of QDs without the need to synthesize new samples.

Ultrafast optical tuning of the PC medium on femtosecond and picosecond timescales has been actively investigated in recent years. This technology has potential applications in novel devices, such as optical switches or signal processing systems. Among the most promising methods for ultrafast tuning are optically induced changes in the refractive index of one or both PC layers through the injection of free charge carriers, the optical Kerr effect, or the optical Stark effect [19, 31–34]. The change in refractive index due to the injection of free charge carriers involves three main contributions: the Burstein–Moss shift of interband

transitions (related to band filling), Drude contributions, and the renormalization of the bandgap (its shrinkage) [19]. We consider the Drude contributions associated with charge carriers excited by one-photon and multiphoton processes, as well as the contribution of the optical Kerr effect to altering the refractive index of the optically dense layer of the one-dimensional PC based on gallium arsenide.



Russian Text	English Text
Собственно-энергетическая поправка, $\langle \delta m_{pc} \rangle$, эВ	Self-energy correction, $\langle \delta m_{pc} \rangle$, eV
Показатель преломления Re(n_h)	Refractive index Re(n_h)

Fig. 1. Quadratic dependence of the self-energy correction $\langle \delta m_{pc} \rangle$ on the refractive index n_h of the substance, composed of one-dimensional PC layers with voids, for various states of the bound electron in a QD: *s*-state ($l = 0$, $m_l = 0$) (curve *2*), p_0-state ($l = 1$, $m_l = 0$) (curve *3*) and $p_{\pm 1}$-state ($l = 1$, $m_l = \pm 1$) (curve *1*). PC parameters: the thickness of the vacuum layers $d_l/T = 1/3$ and the material layers $d_h/T = 2/3$, where $T = 750$ nm is the PC period, and the cutoff parameter $\omega^{kn}_{max} = 10.65$ eV.

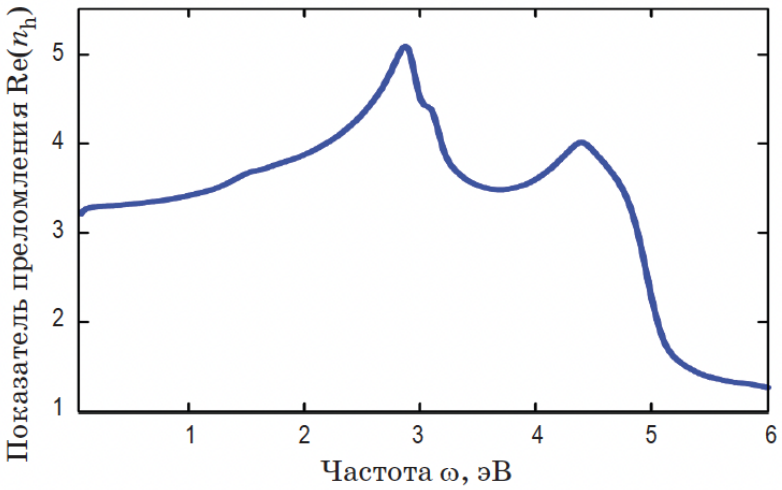
The first contributions occur when the pump beam energy ($\hbar\Omega \approx 1.55$ eV) slightly exceeds the energy of the GaAs electronic bandgap ($E_g = 1.42$ eV at $T = 295$ K) [19]. The contribution of the electro-optic Kerr effect is observed when the pump energy is below $E_g/2$. In this study, a droplet-shaped single quantum dot (QD) made of indium arsenide (InAs), located on the surface of GaAs, was examined. Since the bandgap of InAs is smaller than that of GaAs, the QD forms a three-dimensional potential well for electrons and holes [35]. This study does not consider electron-phonon interactions, which typically occur at cryogenic temperatures. In [19], it was shown that thermally induced changes in the refractive index of GaAs are approximately two to three orders of magnitude smaller than those caused by band filling and Drude contributions. Therefore, thermal effects can be neglected.

We will examine the Drude contributions and the electro-optical Kerr effect on the refractive index of gallium arsenide in more detail. Based on data from [19], we assume that the injection of free charge carriers, which occurs under the action of an optical pulse with a wavelength of $\lambda = 880$ nm and an intensity of $I_\Omega \approx 0.4$ GW/cm^2, results in a change in the refractive index of GaAs, Δn, equal to 0.01. The refractive index n of undisturbed GaAs at a wavelength of $\lambda = 880$ nm (≈ 1.55 eV) is 3.666 (Fig. 2) [36, 37].

The calculated values of the energy transitions in a single InAs QD located on a GaAs layer in a one-dimensional PC, through the mechanism of free charge carrier injection, are shown in Table 1.

Table 1. Corrections to the transition energies of a single InAs QD on a layer of one-dimensional GaAs PC caused by the effect of a change in the electromagnetic mass of an electron. The values of the corrections (Column 2) for GaAs layers undisturbed by the external field, as well as the values of the corrections (Column 3) for the altered refractive index of GaAs layers due to Drude contributions, are shown.

Corrections	Undisturbed GaAs (n ≈ 3.666), MeV	GaAs with Drude Contributions (n ≈ 3.676), MeV	Difference, μeV
Δz_1	1.524	1.473	–51.5
Δz_2	–0.762	0.736	25.7



Russian Text	English Text
Показатель преломления Re(n_h)	Refractive index Re(n_h)
Частота ω, эВ	Frequency ω, eV

Fig. 2. Dispersion function $n_h(\omega)$ for GaAs [36, 37].

The change in the refractive index of gallium arsenide through the Kerr effect is achieved by a probe beam at frequency ω in the presence of a pump beam at frequency Ω. This is expressed as follows:

add rp73, (16)

where $\chi^{(3)}$ denotes the third-order optical susceptibility, and n_2 denotes the nonlinear refractive index. The nonlinear refractive index n_2 for GaAs is 3×10^{-4} cm^2/GW^{-1} [19]. To assess the Kerr effect, we considered the propagation of picosecond pulses with a wavelength of $\lambda = 1900$ nm and an intensity of $I_\Omega \approx 40.4$ GW/cm^2, based on the parameters from [31]. Under the influence of this non-resonant excitation of the one-dimensional GaAs PC medium and a single InAs QD, the change in the refractive index of the optically dense PC layer Δn was 0.024. The value of n for undisturbed GaAs at a wavelength of $\lambda = 1900$ nm (≈ 0.63 eV) was 3.342 (Fig. 2) [36, 37]. This study considers only the real part of the refractive index $\mathrm{Re}(n_h)$ of gallium arsenide, as the imaginary part leads to broadening of the spectral lines. The calculated values of the energy transitions in a single InAs QD, located on a GaAs layer in a one-dimensional PC, through the Kerr effect are shown in Table 2.

Table 2. Corrections to the transition energies of a single InAs QD on a layer of one-dimensional GaAs PC caused by the effect of a change in the electromagnetic mass of an electron. The values of the corrections (Column 2) for GaAs layers undisturbed by the external field, as well as the values of the corrections (Column 3) for the altered refractive index of GaAs layers using the electro-optic Kerr effect, are shown.

Corrections	Undisturbed GaAs (n ≈ 3.342), MeV	GaAs with Kerr Effect (n ≈ 3.366), MeV	Difference, μeV
Δz_1	1.710	1.674	–35.2
Δz_2	–0.855	0.837	17.6

The calculated values of corrections to the transition energies between the *p*- and *s*-states of the electron in an indium arsenide QD situated in a one-dimensional PC made of gallium arsenide and vacuum layers, in the undisturbed case, amounted to units of ~~MeV~~ meV. In contrast, the range of energy changes induced by the injection of free charge carriers and the electro-optic Kerr effect was two orders of magnitude smaller. Due to the quadratic dependence of the self-energy correction $\langle \delta m_{pc} \rangle$ on the refractive index n_h of the optically dense GaAs PC layer, the shift of the transition energies can be greater. This requires a change in n_h, which can be achieved through the injection of free charge carriers and the electro-optic Kerr effect. In these cases, the additional correction shifts to the transition energies of the electron in the indium arsenide QD within the one-dimensional PC medium of gallium arsenide and vacuum layers amounted to tens of μeV. The studied QED effect is enhanced when the PC consists of layers with high optical contrast, as well as a large nonlinear refractive index n_2.

5. CONCLUSION

Using the example of quantum transitions between the *p*- and *s*-states of a hydrogen-like InAs QD placed in the cavities of a one-dimensional GaAs PC, the fundamental possibility of in situ control of the photon frequencies emitted by the QD has been demonstrated. This is achievable through the effect of changing the electromagnetic mass of the electron and by

tuning the refractive index of the PC via the injection of free charge carriers and the electro-optic Kerr effect.

Calculations conducted under the described conditions have shown that the current range of photon energy control available in the experiment is too small for practical applications, and the magnitude of the shift is smaller than what has been observed experimentally in prior studies [38–40]. However, it is important to note that the energy level shift due to the studied QED effect depends quadratically on the refractive index of the material from which the PC is constructed. Therefore, it is expected that this method will be significantly scalable as the optical density of the material increases. Such PCs can be fabricated using metamaterials with a high refractive index [23, 24].

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REFERENCES

1. A. V. Krasheninnikov, L. A. Openov, "Elementary quantum dot gates for single-electron computing," JETP Lett. **64**, 231–236 (1996) [Pis'ma Zh. Eksp. Teor. Fiz. **64**(3), 214–218 (1996)]. https://doi.org/10.1134/1.567180
2. J. L. O'Brien, A. Furusawa, J. Vučković, "Photonic quantum technologies," Nat. Photonics **3**(12), 687–695 (2009). https://doi.org/10.1038/nphoton.2009.229
3. T. Heindel, J. H. Kim, N. Gregersen, et al., "Quantum dots for photonic quantum information technology," Adv. Opt. Photonics **15**(3), 613–738 (2023). https://doi.org/10.1364/AOP.490091
4. M. V. Rakhlin, K. G. Belyaev, S. V. Sorokin, et al., "Single-photon emitter at 80 k based on a dielectric nanoantenna with a CdSe/ZnSe quantum dot," JETP Lett. **108**, 201–204 (2018) [Pis'ma Zh. Eksp. Teor. Fiz. **108**(3), 201–205 (2018)]. https://doi.org/10.1134/S0021364018150109
5. P. Senellart, G. Solomon, and A. White, "High-performance semiconductor quantum-dot single-photon sources," Nat. Nanotechnol. **12**(11), 1026–1039 (2017). https://doi.org/10.1038/nnano.2017.218
6. C. M. Lee, M. A. Buyukkaya, S. Aghaeimeibodi, et al., "A fiber-integrated nanobeam single photon source emitting at telecom wavelengths," Appl. Phys. Lett. **114**(17), 171101 (2019). https://doi.org/10.1063/1.5089907
7. R. Uppu, L. Midolo, X. Zhou, et al., "Quantum-dot-based deterministic photon–emitter interfaces for scalable photonic quantum technology," Nat. Nanotechnol. **16**(12), 1308–1317 (2021). https://doi.org/10.1038/s41565-021-00965-6
8. R. Kh. Gainutdinov, L. Y. Nabieva, A. I. Garifullin, et al., "Strong interaction effects in the emission spectra of a quantum dot coupled to a phonon reservoir," JETP Lett.

114, 188–194 (2021) [Pis'ma Zh. Eksp. Teor. Fiz. **114**(4), 221–227 (2021)]. https://doi.org/10.1134/S0021364021160050

9. C. Livache, B. Martinez, N. Goubet, et al., "A colloidal quantum dot infrared photodetector and its use for intraband detection," Nat. Commun. **10**(1), 2125 (2019). https://doi.org/10.1038/s41467-019-10170-8

10. I. Y. Eremchev, M. Y. Eremchev, and A. V. Naumov, "Multifunctional far-field luminescence nanoscope for studying single molecules and quantum dots: (50th anniversary of the Institute of Spectroscopy, Russian Academy of Sciences)," Phys.-Uspekhi **62**(3), 294–303 (2018) [Usp. Fiz. Nauk **189**(3), 312–322 (2019)]. https://doi.org/10.3367/UFNe.2018.06.038461

11. H. Hosokawa, R. Tamaki, T. Sawada, et al., "Solution-processed intermediate-band solar cells with lead sulfide quantum dots and lead halide perovskites," Nat. Commun. **10**(1), 43 (2019). https://doi.org/10.1038/s41467-018-07655-3

12. A. K. Visheratina, I. V. Martynenko, A. O. Orlova, et al., "Investigation of biocompatible complexes of Mn^{2+}-doped ZnS quantum dots with chlorin e6," J. Opt. Technol. **81**(8), 444–448 (2014) [Opt. Zh. **81**(8), 31–37 (2014)]. https://doi.org/10.1364/JOT.81.000444

13. A. Faraon, A. Majumdar, D. Englund, et al., "Integrated quantum optical networks based on quantum dots and photonic crystals," New J. Phys. **13**(5), 055025 (2011). https://doi.org/10.1088/1367-2630/13/5/055025

14. Y. Yu, W. Xue, E. Semenova, et al., "Demonstration of a self-pulsing photonic crystal Fano laser," Nat. Photonics **11**(2), 81–84 (2017). https://doi.org/10.1038/nphoton.2016.248

15. I. N. Chuprina and A. A. Kalachev, "Generating frequency-bin qubits via spontaneous four-wave mixing in a photonic molecule," Phys. Rev. A **100**(4), 043843 (2019). https://doi.org/10.1103/PhysRevA.100.043843

16. M. Lermer, N. Gregersen, F. Dunzer, et al., "Bloch-wave engineering of quantum dot micropillars for cavity quantum electrodynamics experiments," Phys. Rev. Lett. **108**(5), 057402 (2012). https://doi.org/10.1103/PhysRevLett.108.057402

17. Y. Wei, T. Zhao, B. Yao, et al., "Bright and highly-polarized single-photon sources in visible based on droplet-epitaxial GaAs quantum dots in photonic crystal cavities," Opt. Mater. Express **10**(1), 170–177 (2020). https://doi.org/10.1364/OME.379424

18. R. Kh. Gainutdinov, M. A. Khamadeev, and M. Kh. Salakhov, "Electron rest mass and energy levels of atoms in the photonic crystal medium," Phys. Rev. A **85**(5), 053836 (2012). https://doi.org/10.1103/PhysRevA.85.053836

19. J. P. Mondia, H. W. Tan, S. Linden, et al., "Ultrafast tuning of two-dimensional planar photonic-crystal waveguides via free-carrier injection and the optical Kerr effect," JOSA B **22**(11), 2480–2486 (2005). https://doi.org/10.1364/JOSAB.22.002480

20. S. John and J. Wang, “Quantum optics of localized light in a photonic band gap,” Phys. Rev. B **43**(16), 12772–12789 (1991). https://doi.org/10.1103/PhysRevB.43.12772

21. X. H. Wang, Y. S. Kivshar, and B. Y. Gu, “Giant lamb shift in photonic crystals,” Phys. Rev. Lett. **93**(7), 073901 (2004). https://doi.org/10.1103/PhysRevLett.93.073901

22. R. Kh. Gainutdinov, A. I. Garifullin, M. A. Khamadeev, et al., “Quantum electrodynamics in photonic crystals and controllability of ionization energy of atoms,” Phys. Lett. A **404**, 127407 (2021). https://doi.org/10.1016/j.physleta.2021.127407

23. S. Lee, “Colloidal superlattices for unnaturally high-index metamaterials at broadband optical frequencies,” Opt. Express **23**(22), 28170 (2015). https://doi.org/10.1364/OE.23.028170

24. R. Kim, K. Chung, J. Y. Kim, et al., “Metal nanoparticle array as a tunable refractive index material over broad visible and infrared wavelengths,” ACS Photonics **5**(4), 1188–1195 (2018). https://doi.org/10.1021/acsphotonics.7b01497

25. C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Atom-photon interactions, basic processes and applications (Wiley Interscience, Weinheim, 2004).

26. S. S. Schweber, An introduction to relativistic quantum field theory (Courier, New York, 2011).

27. M. Skorobogatiy and J. Yang, Fundamentals of photonic crystal guiding (Cambridge University Press, New York, 2009).

28. M. Richter, A. Carmele, A. Sitek, et al., “Few-photon model of the optical emission of semiconductor quantum dots,” Phys. Rev. Lett. **103**(8), 087407 (2009). https://doi.org/10.1103/PhysRevLett.103.087407

29. Y. I. Diakite, Y. Malozovsky, C. O. Bamba, et al., “First principle calculation of accurate electronic and related properties of zinc blende Indium Arsenide (zb-InAs),” Materials **15**(10), 3690 (2022). https://doi.org/10.3390/ma15103690

30. S. Flugge, Practical quantum mechanics (Springer, New York, 2011).

31. X. Hu, Q. Zhang, Y. Liu, et al., “Ultrafast three-dimensional tunable photonic crystal,” Appl. Phys. Lett. **83**(13), 2518–2520 (2003). https://doi.org/10.1063/1.1615309

32. A. Hartsuiker, P. J. Harding, Y. R. Nowicki-Bringuier, et al., “Kerr and free carrier ultrafast all-optical switching of GaAs/AlAs nanostructures near the three photon edge of GaAs,” J. Appl. Phys. **104**(8), 083105 (2008). https://doi.org/10.1063/1.3000098

33. K. Morita, T. Takahashi, T. Kanbara, et al., “Large optical Kerr signal of GaAs/AlAs multilayer cavity with InAs quantum dots embedded in strain-relaxed barriers,” Phys. E **42**(10), 2505–2508 (2010). https://doi.org/10.1016/j.physe.2009.12.035

34. K. Rivoire, S. Buckley, A. Majumdar, et al., “Fast quantum dot single photon source triggered at telecommunications wavelength,” Appl. Phys. Lett. **98**(8), 083105 (2011). https://doi.org/10.1063/1.3556644

35. A. A. Ramsay, “A review of the coherent optical control of the exciton and spin states of semiconductor quantum dots,” Semicond. Sci. Technol. **25**(10), 103001 (2010). https://doi.org/10.1088/0268-1242/25/10/103001

36. D. Aspnes, S. Kelso, R. Logan, et al., “Optical properties of AlxGa1–xAs,” J. Appl. Phys. **60**(2), 754–767 (1986). https://doi.org/10.1063/1.337426

37. T. Skauli, P. S. Kuo, K. L. Vodopyanov, et al., “Improved dispersion relations for GaAs and applications to nonlinear optics,” J. Appl. Phys. **94**, 6447–6455 (2003). https://doi.org/10.1063/1.1621740

38. H. Kim, T. C. Shen, D. Sridharan, et al., “Magnetic field tuning of a quantum dot strongly coupled to a photonic crystal cavity,” Appl. Phys. Lett. **98**(9), 091102 (2011). https://doi.org/10.1063/1.3562344

39. S. Aghaeimeibodi, C. M. Lee, M. A. Buyukkaya, et al., “Large stark tuning of InAs/InP quantum dots,” Appl. Phys. Lett. **114**(7), 071105 (2019). https://doi.org/10.1063/1.5082560

40. R. Katsumi, Y. Ota, A. Osada, et al., “In situ wavelength tuning of quantum-dot single-photon sources integrated on a CMOS-processed silicon waveguide,” Appl. Phys. Lett. **116**(4), 041103 (2020). https://doi.org/10.1063/1.5129325

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