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Multi-scale hierarchy from multidimensional gravity

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ABSTRACT

We discuss the way of solving the hierarchy problem. We show that starting at the Planck scale, the three energy scales — inflationary, electroweak and the cosmological ones can be restored. A mechanism for generating small parameters that leads to a successful solution of the problem is proposed. The tools involved in the process are $f(R)$ gravity and inhomogeneous extra dimensions. Slow rolling of a space domain from the Planck scale down to the inflationary one gives rise to three consequences: an infinite set of causally disconnected domains (pocket universes) are nucleated; quantum fluctuations in each domain produce a variety of fields and an extra-dimensional metric distribution; these distributions are stabilized at a sufficiently low energy scale.

1. Introduction

Assuming that the Universe has been formed at the Planck scale, it is naturally implied that its initially formed parameters are of the order of the same scale. The essence of the Hierarchy problem is the question: Why are the observable low-energy physical parameters so small as compared to those of the Planck scale? How did Nature manage to decrease the parameter values so substantially?

There are at least four important energy scales during evolution of the Universe: the Planck scale ($\sim 10^{19}$ GeV) at which our Universe cannot be described by classical laws; the inflationary scale ($\sim 10^{13}$ GeV) where our horizon has appeared, the electroweak scale ($\sim 10^{2}$ GeV), and the cosmological scale specified by the cosmological constant ($\sim 10^{-61}$ GeV^2) (CC).

According to the inflationary paradigm, the physical laws are formed at high energies [1,2], where the Lagrangian structure is yet unknown. Therefore, physics has been established at an energy scale M between the inflationary scale $E_I \sim 10^{13}$ GeV and the Planck scale $E_P \sim 10^{19}$ GeV, see [3,4] in this context. We study the way of physical parameter reduction at the mentioned scales, which are below the initial scale M.

In this paper, we invoke the idea of multidimensional gravity which is a widely used tool for obtaining new theoretical results [5–9]. The paper [10] uses warped geometry to solve the small cosmological constant problem. Multidimensional inflation is discussed in [11–13] where it was supposed that an extra-dimensional metric g_n is stabilized at a high-energy scale. Stabilization of extra space as a pure gravitational effect has been studied in [14,15], see also [16].

The present research is also based on nonlinear $f(R)$ gravity. The interest in $f(R)$ theories is motivated by inflationary scenarios starting with Starobinsky's paper [17]. At present, $f(R)$ gravity is widely discussed [18,19], leading to a variety of consequences, in particular, the existence of dark matter [20,21]. Including a function of the Ricci scalar, $f(R)$, is the simplest extension of general relativity. In the framework of such an extension, many interesting results have been obtained. Some viable $f(R)$ models in 4D space that satisfy the observational constraints are proposed in [22–26].

An application of nonlinear gravity to the description of the cosmological constant has been done in [27]. As shown there, this approach suffers from overproduction of scalar particles. The authors of [28–30] considered a class of $f(R)$ models operating over a wide range of distances.

The idea that the Lagrangian parameters can be considered as some functions of a field has been widely used since Schwinger's paper [31]. Such fields can be involved in the classical equations of motion together with the "main" fields or treated as background fields. The latter were applied for fermion localization on branes [32–34], gauge field localization [35], extensions of gravity in a scalar-tensor form (with $f(\phi)R$) [36] and so on. In this paper, we show that a self-gravitating scalar field can serve as a reason for the emergence of small parameters.

As a mathematical tool, we use the Wilsonian approach [37] technique, a well-known method for theoretical studies of the energy

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scale. It is achieved by sequentially integrating the Euclidean action over a small slice of the momentum interval Δk_E. The renormalization group equations thus obtained are widely used in this concern [38]. The relations between low-energy parameter values and high-energy ones are discussed in [68]. Also, quantum fluctuations could modify the same form of the Lagrangian [69,70].

The inclusion of a compact extra space into consideration complicates the procedure. Indeed, we cannot choose an arbitrarily small momentum interval due to the energy level discreteness. For example, if a size is quite small, $\Delta k_E < 1/r$, r being the scale of extra dimensions, then this momentum interval does not contain energy levels at all. A possible way to overcome this difficulty is discussed in [48], where the truncated Green functions

$$G_T(Z,Z') \equiv \sum_{N\in\mathcal{N}} \frac{Y_N(Z)Y_N(Z')^*}{\lambda_N}$$

were introduced. Here $Y_N(Z)$ is a subset of $n+4$-dimensional eigenfunctions. The coordinates Z describe both 4D space and a compact extra space. It allows for approximately calculating the parameters at low energies. As a result, quantum corrections caused by a scalar field are proportional to its self-coupling. This means that such quantum effects cannot be responsible for reducing the parameter values by many orders of magnitude, from the Planck scale to the electroweak scale. The classical mechanism discussed in this paper was elaborated just for this aim. The procedure of quantum renormalization is a necessary and unavoidable element that leads to fine tuning of the physical parameters at low energies.

7. Matter localization around a singularity

In this section, we briefly discuss a possible extension of our approach to show that matter concentrates near singularities, forming a kind of thick branes. In general, it is assumed here that matter is distributed throughout the extra dimensions as in the Universal Extra Dimensional approach [71,72]. At the same time, there is another point worth discussing. Indeed, we see from Fig. 1 that there are two points where the extra metric is singular or has sharp peaks. They could indicate the formation of branes if the extra space is large enough and if matter is concentrated in a close neighborhood of these peaks (certainly assuming that the formal infinities are somehow suppressed by quantum effects). The structure of such a brane should be rather nontrivial because of the presence of a possible singularity. This direction may be developed in the future.

As shown in Appendix A, matter is localized around both 'poles', as it should be in a brane world. It opens a door for developing a mechanism of strong reduction of the initial parameter values. For example, an interaction term of the form

$$\kappa \int d^D Z \sqrt{|g_D|}\chi(z)\bar{\psi}(z)\psi(z)$$

contains the overlapping integral

$$I_{\text{overlap}} \equiv \int d^n y \sqrt{|g_n|}\chi(y)\bar{\psi}(y)\psi(y)$$

over the extra dimensions which could be arbitrarily small if the fields $\chi(y)$ and $\psi(y)$ are localized near different branes. It leads to the coupling constant renormalization

$$\kappa \to \kappa' = \kappa I_{\text{overlap}} \ll \kappa.$$

We will leave this idea for future studies.

8. Conclusion

This paper discusses the reduction mechanism of the physical parameters values defined at high energies to those now observed. Starting from a unified Lagrangian at high energies, we have succeeded in fitting the physical parameters describing different physical phenomena — inflation, the Higgs field and the cosmological constant. The flexible extra metric is a necessary tool for a successful solution of the problem.

The set of small parameters is formed in the following way. Slow rolling of a spatial domain from a sub-Planckian scale down to the inflationary one gives rise to several consequences: (1) nucleation of an infinite set of causally disconnected domains (pocket universes), (2) quantum fluctuations in each domain produce a variety of fields and an extra-space metric distribution, (3) these distributions are stabilized when the energy scale is low enough. Self-gravitating (scalar) fields do not necessarily settle at states with minimum energy. On the contrary, e.g., the boson stars activity [63] is based on the fact that self-gravitating scalar fields can settle at a continuum set of static states. There are states with arbitrarily small amplitudes among them. These states are formed in a small but finite set of universes. As a result, a small but nonzero measure of different universes contains small effective parameters that are applied here to solve the Hierarchy problem at three energy scales.

Attempts to experimentally test the extra dimensions paradigm are repeatedly being made. Traces of Large Extra Dimensions are searched for on cosmological scales [73] and in colliders [74]. Non-compact extra dimensions affect the propagation of gravitational waves at cosmological distances [75]. At the same time, attempts to find traces of extra dimensions run into difficulties if they are compact and stabilize at energies above the inflationary one, as happens in the case discussed here. One of the possible directions in this case is to study the change in inflationary parameters as the Hubble parameter decreases.

We hope that the mechanism elaborated here will open a way to fix other physical parameters observed at low energies, starting from a unified Lagrangian at high energies. The mechanism developed should be accompanied by a renormalization group analysis aimed at correction of the initial parameter values.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Geodesics in extra dimensions

Consider the motion of classical particles in a gravitational background described by the metric

$$ds^2 = \mathrm{e}^{2\gamma(u)}(dt^2 - dx^2 - dy^2 - dz^2) - du^2 - r(u)^2(d\xi^2 + \sin^2\xi\, d\psi^2) \quad (48)$$

where the functions $r(u)$ and $\gamma(u)$ are solutions of Eqs. (6), (8), (9), (10) for parameters indicated in Fig. 1, with the exception of $R(0) \simeq 0.004$ and $H = 0$, as was done in Section 4.

In this background, the geodesic equations have the form

$$\ddot{t} + 2\,\dot{t}\,\gamma'\,\dot{u} = 0, \quad (49)$$

$$\ddot{x} + 2\,\dot{x}\,\gamma'\,\dot{u} = 0, \quad \ddot{y} + 2\,\dot{y}\,\gamma'\,\dot{u} = 0, \quad \ddot{z} + 2\,\dot{z}\,\gamma'\,\dot{u} = 0, \quad (50)$$

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