

$$\vec{F} = m\vec{a}$$

S.V. Efimov
A.N. Turanov

**PRACTICAL WORK
MEDICAL PHYSICS**

**PART 1. MECHANICS
PART 2. MOLECULAR PHYSICS**

$$pV = \frac{m}{M}RT$$

**KAZAN FEDERAL UNIVERSITY
INSTITUTE OF PHYSICS
DEPARTMENT OF MEDICAL PHYSICS**

**S.V. EFIMOV
A.N. TURANOV**

**PRACTICAL WORK
MEDICAL PHYSICS**

**Part 1. Mechanics
Part 2. Molecular physics**

Study aid

**For English-speaking students of medical, biomedical
and pharmaceutical fields of study**

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Authors:

Senior Researcher, Research Laboratory of Magnetic Radio Spectroscopy
and Quantum Electronics, PhD **S.V. Efimov**;
Professor of the Department of Medical Physics, PhD, Doctor of Technical Sciences
A.N. Turanov

Reviewers:

Associate Professor, Head of the Department of medical and biological physics with
informatics and medical equipment of the Kazan State Medical University, PhD
R.S. Gimatdinov;
Assistant Professor, School of Biomedical Engineering, The University of Western
Ontario, Associate Scientist, Lawson Health Research Institute, PhD **A. Ouriadov**

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Collection of descriptions of laboratory work “Practical work. Medical physics.
Part 1. Mechanics. Part 2. Molecular physics” is intended for students of foreign
groups of medical specialties of the University. For each work, a brief overview,
installation manuals, step-by-step instructions, and a list of questions for self-study
are provided.

UDC 530
LBC 22.3

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We welcome any feedback from students and professors, especially concerning errors or deficiencies. Please, feel free to contact us:

Sergey Efimov and Alex Turanov
sergej.efimov@kpfu.ru anturanov@kpfu.ru

Introduction

Standards and units

Physics is an experimental science. Experiments require measurements, and we usually use numbers to describe the results of measurements. Any number that is used to describe a physical phenomenon quantitatively is called a physical quantity. For example, two physical quantities that describe you are your weight and height. Some physical quantities are fundamental, so we can define them only by describing how to measure them. Such a definition is called an operational definition. Some examples are measuring a distance by using a ruler, and measuring a time interval by using a stopwatch. In other cases we define a physical quantity by describing how to calculate it from other quantities that we can measure. Thus, we might define the average speed of a moving object as the distance travelled divided by the time of travel.

When we measure a quantity, we always compare it with some gold standard. The metre is a unit of distance, and the second is a unit of time.

To make accurate, reproducible measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations. The system of units used by scientists and engineers around the globe is commonly called “the metric system,” but since 1960 it has been known officially as the International System, or SI (the abbreviation for its French name, *Système International*).

The definitions of the basic units of the metric system have evolved over the years. When the metric system was established in 1791 by the French Academy of Sciences, the metre was defined as one ten-millionth of the distance from the North Pole to the equator. The second was defined as the time required for a pendulum one metre long to swing from one side to the other. These definitions were cumbersome and hard to duplicate precisely, and by international agreement they have been replaced with more refined definitions.

Time. The present standard, adopted in 1967, is very precise. It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the caesium atom. When bombarded by microwaves of precisely the proper frequency, caesium atoms undergo a transition from one of these states to the other. One second is defined as the time required for 9,192,631,770 cycles of this radiation.

Length. In November 1983 the length standard was changed. The speed of light in a vacuum was defined to be precisely 299,792,458 m/s. The metre is defined to be consistent with this number and with the above definition of the second. Hence the new definition of the metre is the distance that light travels in a vacuum in $1/299,792,458$ second. This provides a much more precise standard of length than the one based on a wavelength of light.

Mass. The standard of mass, the kilogram, is defined to be the mass of a particular cylinder of platinum-iridium alloy. That cylinder is kept at the International Bureau of Weights and Measures at Sévres, near Paris. An atomic standard of mass would be more fundamental, but at present we cannot measure masses on an atomic scale with as much accuracy as on a macroscopic scale.

Unit prefixes

Once we have defined the fundamental units, it is easy to introduce larger and smaller units for the same physical quantities. In the metric system these other units are always related to the fundamental units by multiples of 10 or 1/10. Thus, one kilometre (1 km) is 1000 metres, and one centimetre (1 cm) is 1/100 metre. We usually express multiples of 10 or 1/10 in exponential notation: $1000 = 10^3$, $1/1000 = 10^{-3}$, and so on. With this notation, $1 \text{ km} = 10^3 \text{ m}$ and $1 \text{ cm} = 10^{-2} \text{ m}$.

The names of the additional units are derived by adding a prefix to the name of the fundamental unit. Here are several examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time.

TIME

$$1 \text{ nanosecond} = 1 \text{ ns} = 10^{-9} \text{ s}$$

$$1 \text{ microsecond} = 1 \mu\text{s} = 10^{-6} \text{ s}$$

$$1 \text{ millisecond} = 1 \text{ ms} = 10^{-3} \text{ s}$$

LENGTH

$$1 \text{ nanometre} = 1 \text{ nm} = 10^{-9} \text{ m}$$

$$1 \text{ micrometre} = 1 \mu\text{m} = 10^{-6} \text{ m}$$

$$1 \text{ millimetre} = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$1 \text{ centimetre} = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$1 \text{ kilometre} = 1 \text{ km} = 10^3 \text{ m}$$

MASS

$$1 \text{ microgram} = 1 \mu\text{g} = 10^{-9} \text{ kg}$$

$$1 \text{ milligram} = 1 \text{ mg} = 10^{-6} \text{ kg}$$

$$1 \text{ gram} = 1 \text{ g} = 10^{-3} \text{ kg}$$

$$1 \text{ kilogram} = 1 \text{ kg} = 10^3 \text{ grams} = 10^3 \text{ g.}$$

Uncertainty and significant figures

Measurements always have uncertainties. The distinction between two measurements is in their uncertainty. The uncertainty is also called the error, because it indicates the maximum difference there is likely to be between the measured value and the true value. The uncertainty or error of a measured value depends on the measurement technique used.

We often indicate the accuracy of a measured value meaning, how close it is likely to be to the real value – by writing the number, the symbol \pm , and a second number indicating the uncertainty of the measurement. If the diameter of a steel rod is given as 56.47 ± 0.02 mm, this means that the real value is unlikely to be less than 56.45 mm or greater than 56.49 mm. The degree of this likelihood (confidence level) is usually stipulated beforehand. In a commonly used shorthand notation, the number 1.6454(21) means 1.6454 ± 0.0021 . The numbers in parentheses show the uncertainty in the final digits of the main number.

We can also express accuracy in terms of the maximum likely

fractional error or percent error (also called fractional uncertainty and percent uncertainty). A resistor labelled “47 ohms \pm 10%” likely has a factual resistance differing from 47 ohms but no more than 10% from 47 ohms, that is, roughly 5 ohms. Thus, the resistance is between 42 and 52 ohms.

In many cases the uncertainty is not stated explicitly. Instead, the uncertainty is stated by a number of the meaningful digits, or significant figures, of the measured value. For example, a thickness of smartphone is 2.91 mm, which is three significant figures. This means that the first two digits are accurate, while the third digit is uncertain. The last digit is in the hundredths place, so the uncertainty is about 0.01 mm. Two values with the same number of significant figures may have different uncertainties; a distance given as 137 km also has three significant figures, but the uncertainty is about 1 km.

If one uses numbers with uncertainties to compute another numbers, the computed numbers also have uncertainties. It is important to keep in mind when comparing a number obtained from measurements with the theoretical prediction. Suppose, one wants to verify the value of π , by taking the ratio of a circle perimeter to a diameter. The correct value of this ratio to ten digits is 3.141592654. To make this calculation, one should draw a large circle and measure its diameter and circle perimeter to the nearest millimetre. Assume one measured the following values 270 mm and 848 mm for the circle diameter and circle perimeter, respectively. A calculator helps us to take the ratio between two numbers and obtain 3.140740741. Does this value agree with the expected π value or not really?

Firstly, the last seven digits in this number are meaningless; they imply a smaller uncertainty than the uncertainty of the circle diameter and perimeter measurements. When values are multiplied or divided, the number of significant figures in the result can be no greater than in the number with the fewest significant figures. For example: $3.1416 \times 2.34 \times 0.58 = 4.3$. Obtained circle diameter and perimeter values had three significant figures, so the calculated value of π , equal to

$(424 \text{ mm})/(135 \text{ mm})$, should have only three significant figures. It should be rounded to 3.14. Within the limit of three significant figures, this value does agree with the expected π value.

Secondly, when one adds or subtracts numbers, it's the position of the decimal point matters, not a number of the significant figures. For example, $123.62 + 8.9 = 132.5$. Although, 123.62 has an uncertainty of about 0.01, 8.9 has an uncertainty of about 0.1. So, their sum has an uncertainty of about 0.1 and should be written as 132.5, not 132.52.

Finally, let's note that precision is not the same as accuracy. A cheap digital watch that says the time is 10:35:17 A.M. is quite precise (the time is given to the second), but if the watch runs several minutes slow, then this value isn't very accurate. On the other hand, a grandpa clock might be very accurate (that is, display the correct time), but if the clock has no second hand, it isn't very precise. A high-quality measurement is always precise and accurate same time.

Estimates and orders of magnitude

We have stressed the importance of knowing the accuracy of numbers that represent physical quantities. But even a very crude estimate of a quantity often gives us useful information. Sometimes we know how to calculate a certain quantity but have to guess at the data we need for the calculation. Or the calculation might be too complicated to carry out exactly, so one can make some rough approximations. In either case the result is a guess, but such a guess can be useful even if it is uncertain by a factor of two, ten, or more. Such calculations are known as an order-of-magnitude estimates.

Vectors

Some physical quantities, such as time, temperature, mass, density, and electric charge, can be expressed by a single value with a unit of measurement. But many other important quantities have a direction associated with them and cannot be described by a single value. Such

quantities play an essential role in many of the central topics of physics, including motion and the phenomena of electricity and magnetism. A simple example of a quantity with direction is the motion of an airplane. To describe such motion completely, one should state not only how fast the airplane is moving, but also in what direction. A velocity of the airplane combined with its direction of motion together constitute a quantity called velocity. Another example is force, which in physics means a push or pull exerted on a body. Giving a complete description of a force means describing both how hard the force pushes or pulls on the body and the direction of a push or pull.

When a physical quantity is described by a single number, we call it a scalar quantity. In contrast, a vector quantity has both a magnitude and a direction in space. Calculations with scalar quantities use the operations of ordinary arithmetic. However, combining vectors requires a different set of operations.

To understand more about vectors and how they combine, we start with the simplest vector quantity, displacement. Displacement is simply a change in position of a point. (The point may represent a particle or a small body.) Fig. 1 (left) represents the change of position from point P_1 to point P_2 , by a line from P_1 to P_2 , with an arrowhead at P_2 , showing the direction of motion. Displacement is a vector quantity because one should state not only how far the particle moves, but also in what direction.

One represents a vector quantity such as displacement by a single letter, such as \vec{A} in Fig. 1 (left). When drawing any vector, one always draws a line with an arrowhead at its tip. The length of the line shows the vector's magnitude, and the direction of the line shows the vector's direction. Displacement is always a straight-line segment, directed from the starting point to the end point, even though the actual path of the particle may be curved. Note that a displacement does not relate directly to the total distance travelled.

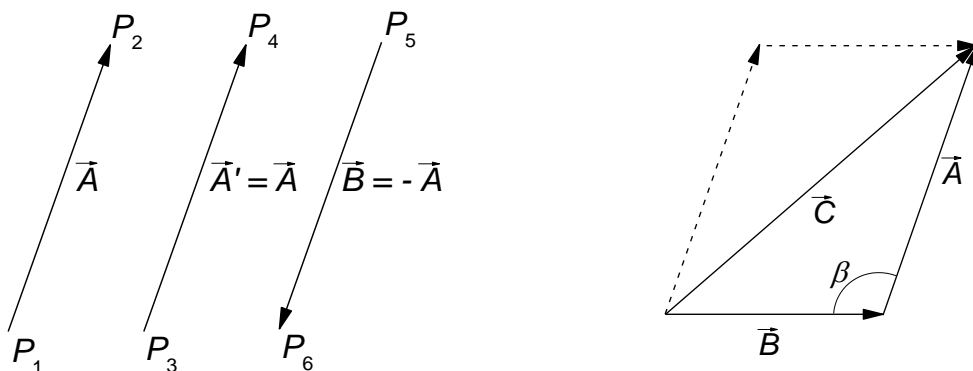


Figure 1. Vectors and vector addition

If two vectors have the same direction, they are parallel. If they have the same magnitude and the same direction, they are equal, no matter where they are located in space. The vector \vec{A}' from point P_3 to point P_4 has the same length and direction as the vector \vec{A} from P_1 to P_2 . These two displacements are equal, even though they start at different points. We write this as $\vec{A} = \vec{A}'$. Two vector quantities are equal only when they have the same magnitude and the same direction.

The vector \vec{B} in Fig. 1 (left), however, is not equal to \vec{A} because its direction is opposite to that of \vec{A} . One defines the negative vector as the vector having the same magnitude as the original vector but the opposite direction. The negative vector \vec{A} is denoted as $-\vec{A}$. Thus, the relation between \vec{A} and \vec{B} of Fig. 1 (left) may be written as $\vec{A} = -\vec{B}$. When two vectors \vec{A} and \vec{B} have opposite directions, whether their magnitudes are the same or not, they are antiparallel.

One represents the magnitude of a vector quantity (its length in the case of a displacement vector) by the same letter used for the vector, in *italic type* but with no arrow on top. An alternative notation is the vector symbol with vertical bars on both sides: (magnitude of \vec{A}) = $A = |\vec{A}|$. By definition the magnitude of a vector quantity is a scalar quantity (a number) and is always positive.

Vector addition

Suppose a particle undergoes a displacement \vec{B} , followed by a second displacement \vec{A} (Fig. 1 (right)). The final result is the same as if the particle had started at the same initial point and undergone a single displacement \vec{C} , as shown. It calls displacement \vec{C} the vector sum. One expresses this relationship symbolically as $\vec{C} = \vec{B} + \vec{A}$. To conduct the vector addition one places the tail of the second vector at the head, or tip, of the first vector (Fig. 1 (right)).

$$C^2 = A^2 + B^2 - 2AB \cos \beta.$$

To add more than two vectors, one should start with finding of the vector sum of any two vectors, then add this vectorially to the third, and so on. A vector quantity such as a displacement can be multiplied by a scalar quantity (an ordinary number). The displacement $2\vec{A}$ is a displacement (vector quantity) in the same direction as the vector \vec{A} but twice as long. The scalar quantity used to multiply a vector may also be a physical quantity having units.

Projections of vectors

To define the projections of a vector, one begins with a rectangular (Cartesian) coordinate system (Fig. 2). We then draw the vector \vec{A} , considering with its tail at O , the origin of the coordinate system. One can represent any vector lying in the xy -plane as the sum of a vector parallel to the x -axis and a vector parallel to the y -axis. These two vectors are labelled \vec{A}_x and \vec{A}_y in the figure, they are called the components of vector \vec{A} , and their vector sum is equal to \vec{A} . By definition, each component vector lies along a coordinate-axis direction. Thus, one needs a single number to describe each component, since their directions are fixed by the directions of the system axes. When the component \vec{A}_x points in the positive x -direction, it defines the number A_x . One defines the number A_y , in the same way. These two numbers A_x and A_y are called the projections of \vec{A} on

the x- and y-axes respectively.

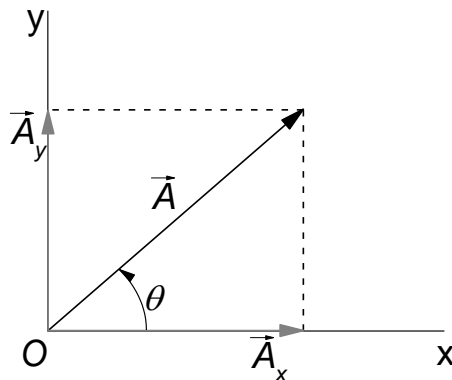


Figure 2. Components and projections of a vector

One can calculate the projections of the vector \vec{A} if based on the magnitude A and its direction. Let's describe the direction of a vector by its angle relative to positive x -axis, and the angle between vector \vec{A} and the positive x -axis is θ . If θ is measured in this way, then from the definition of the trigonometric functions,

$$A_x = A \cdot \cos \theta \text{ and } A_y = A \cdot \sin \theta.$$

How to draw a Report on the results of your lab work

1. Write the title of the lab, describe the tasks to solve.
2. Attach the graphics.
3. Make conclusions corresponding to the goals of the work. Explain the shape of each observed dependence (linear, quadratic) based on the theory and formulas. For example: Describe how the path, velocity, and acceleration change as the weight load increases. Determine the mean velocity on the whole trolley's path for each weight used.

List of literature

In compiling this digest, we used the following list:

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Part 1. MECHANICS

111. Straight-line motion

Objective

Determine the dependence of the coordinate, velocity, and acceleration on time in the case of straight-line motion with constant acceleration.

Task

Analyse the obtained results, compare them with the theory.

The motion is the way that matter exists. In a common sense, motion includes any change, all processes occurring in the Universe, from mechanical motion to the thinking process.

The mechanical motion is the simplest form of motion. It is a change of the mutual disposition of bodies or their parts in space with time. The field of science studying basic laws of mechanical motion and equilibrium of material bodies is called mechanics. Classical mechanics considers a motion of absolutely rigid bodies (which are not deformed when moving) with speeds negligible compared to the speed of light. The motion of a mass point is an example of the simplest mechanical motion.

Mass point is a body whose size and shape are unimportant. If the sizes of a body are small compared to the way which is passed and to the distance to other bodies considered in a given task, then this body may be considered as a mass point.

Motion of material bodies takes place in space and time.

Space and time are forms of existence and changing the states of the matter; they are inherent attributes of the matter. To describe the motion, one needs a frame of reference. The reference frame should include a reference body (a real or abstract one), a system of coordinates bound to that body, and a method of measuring time. For example,

the position of a mass point in any instance may be fixed in a Cartesian frame of reference (Fig. 111.1).

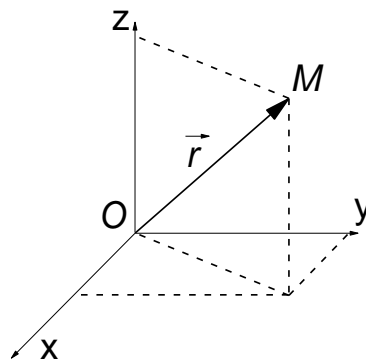


Figure 111.1. Cartesian frame

As the mass point moves, its coordinates change with time:

$$\begin{aligned} x &= f(t), \\ y &= g(t), \\ z &= h(t). \end{aligned} \tag{111.1}$$

The motion of the point is fully described, if equations (111.1) are known. Three equations can be rewritten in a brief vector form:

$$\vec{r} = \vec{r}(t), \tag{111.2}$$

where \vec{r} is the radius vector connecting the origin of the reference frame with the moving mass point.

Any mechanical motion has a relative nature. A particular way of motion, defined by the trajectory, way, displacement, velocity and acceleration, depends on the choice of the reference frame. Let us clarify it by a few examples.

Imagine a ship travelling uniformly along a straight line on a river. A material body detaches from the top of its mast and falls down. In the reference frame bound to the ship (in other words, to an observer on the ship's deck) the trajectory of the body is a vertical line, but for an observer staying on the river bank the trajectory of the same body looks like a curved line (a parabola).

Imagine also a man sitting in a carriage in a moving train. In the reference frame bound to the carriage, the speed of the man is zero, whereas with respect to the railway the speed of the man equals the speed of the train.

Ways covered by a body may also be different in different reference frames, and so on.

Kinematics

Kinematics is a part of mechanics studying the laws of motion without considering the reasons which makes the bodies move.

Physical notions characterising the kinematics of a mass point

The trajectory is a line passed in space by a moving mass point. Depending on the shape of the trajectory, one can speak about straight-line (rectilinear) or curvilinear motion. Path (distance) is the length of a trajectory segment, which was passed by the mass point in a certain time range. Displacement is the vector connecting the initial and current positions of the mass point.

In Fig. 111.2 A stands for the initial position (at $t = 0$), B is its position at time t , the arc AaB is the trajectory, the length of the arc AaB is the distance (ΔS), and $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ is the displacement.

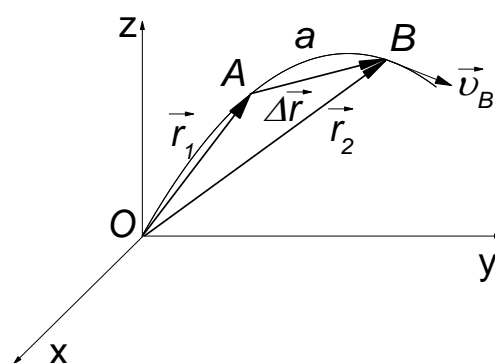


Figure 111.2. Trajectory, path, displacement, and velocity

Average velocity is a vector value characterising the rate and direction of the motion of the mass point:

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}.$$

Instantaneous velocity equals the first derivative of the displacement with respect to time:

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

It follows that in the case of curvilinear (not straight-line) motion, the velocity vector is tangent to the trajectory in the corresponding point (slope of the line on graph). See \vec{v}_B on Fig. 111.2.

Acceleration is a vector value describing the rate at which the velocity changes (in a general case, both in the absolute value and direction). The acceleration vector is the first derivative of the velocity with respect to time:

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

Evidently, acceleration of straight-line motion equals the first derivative of the absolute value of the speed with respect to time.

Uniform straight-line motion

Motion along a straight line at a constant speed is called uniform and rectilinear. In this case $\Delta S = v \cdot \Delta t$ or $S = v \cdot t$.

The unit in which the speed is measured is metre per second (m/s).

The dependence of the speed $v(t)$ can be plotted as a diagram with time corresponding to the horizontal (x -coordinate) axis and the speed shown by the vertical axis. In the case of the uniform motion, $v = \text{const}$, and the plot looks like a horizontal line parallel to the x -coordinate axis (Fig. 111.3 (left)).

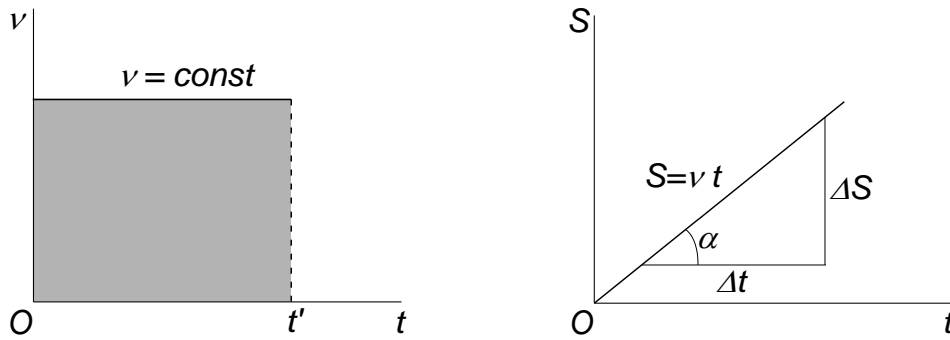


Figure 111.3. Speed and distance in the case of uniform straight-line motion as a function of time

The area of the figure bounded by the speed plot, horizontal time axis, and two vertical lines corresponding to the time moments of beginning and end of the motion, is equal to the distance passed by the mass point in the corresponding time range.

Fig. 111.3 (right) is a distance plot (the horizontal axis shows the time; the vertical axis, the path). It is a straight line starting at the origin O ($t = 0$, $S = 0$). The slope of the line is speed: $\tan \alpha = \Delta S / \Delta t = v$.

Straight-line motion with a constant acceleration

When the velocity of a mass point changes with time, the motion is called variable motion. Uniformly variable motion is characterised by some constant value of the acceleration a , and in the case of straight-line motion $dv = a \cdot dt$, and

$$v = \int a \cdot dt = a \cdot t + C .$$

The integration constant C can be found from the boundary condition: $v = v_0$ at $t = 0$, therefore, $C = v_0$. Finally, $v = v_0 + a \cdot t$.

Acceleration (absolute value) of a uniformly variable straight-line motion is equal to the change in the speed per time unit. Positive sign of a corresponds to the accelerated motion ($v > v_0$); negative, to the decelerated motion ($v < v_0$). Unit of measurement for acceleration is m/s^2 .

Now we can calculate the path:

$$S = \int_0^t v \cdot dt = \int_0^t (v_0 + a \cdot t) \cdot dt = v_0 \cdot \int_0^t dt + a \cdot \int_0^t t \cdot dt = v_0 \cdot t + \frac{a \cdot t^2}{2} .$$

Let us consider now the diagrams of the speed and path of straight-line motion with a constant acceleration (deceleration). Analytical dependence of the speed on time is the equation of a straight line which crosses the vertical axis at the value of v_0 (Fig. 111.4). The slope of the line is $a = \tan \alpha$.

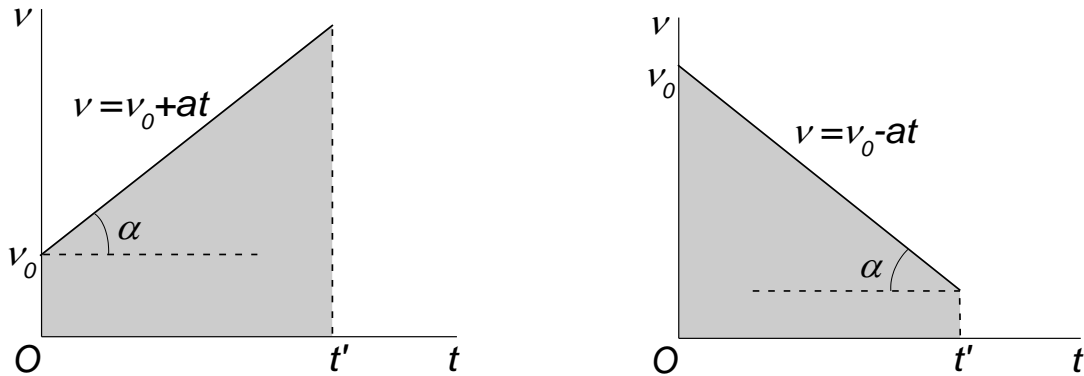


Figure 111.4. Speed dependence of motion with a constant acceleration (deceleration)

The shaded area in Fig. 111.4 shows the path covered by the mass point for motion time t' . The area of this trapezium is

$$S = \frac{(v_0+v) \cdot t'}{2} = \frac{(v_0+v_0+a \cdot t') \cdot t'}{2} = v_0 \cdot t' + \frac{a \cdot t'^2}{2} .$$

This is the second order polynomial, and its plot is a curved line (a parabola). In the case of accelerated motion this parabola has its convex downwards; in the case of decelerated motions, upwards (Fig. 111.5).

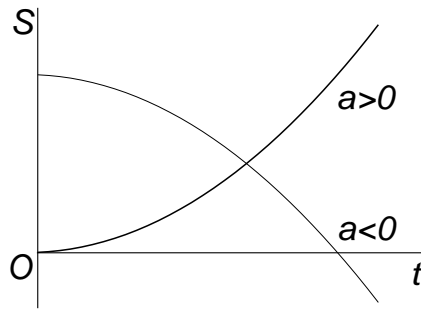


Figure 111.5. Coordinate dependence of motion with a constant acceleration (deceleration)

Dynamics

Dynamics is the part of mechanics studying motion of bodies and its reasons (interaction between the bodies). Dynamics is based on three laws of Newton.

Newton's first law states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.

The second law: acceleration gained by a body as a result of some forces acting on it is proportional to the geometrical (vector) sum of these forces and reciprocal mass of this body.

$$\vec{a} = \frac{\sum_{i=1}^n \vec{F}_i}{m} = \frac{\vec{F}_R}{m},$$

here \vec{a} is the acceleration of the body; m is its mass; $\sum_{i=1}^n \vec{F}_i$ is the vector sum of all external forces acting on the body; \vec{F}_R is the symbol for the net force. Note that the direction of acceleration is the same as the direction of the net force.

Force is a quantitative description of the interaction between two bodies or between a body and its environment (fields). The SI unit of the magnitude of force is the newton (N). Mass characterises the inertia properties of the body (kg) in this case.

The third law states that for every action (force) in nature, there is an equal and opposite reaction. In other words, if object A exerts a force on object B, then object B also exerts an equal force on object A. Notice that the forces act on different objects. Mathematically, this can be written as:
 $\vec{F}_1 = -\vec{F}_2.$

Experimental setup

1. Rails
2. CASSY
3. Trolley
4. Fixed pulley with passage light sensor
5. Rope
6. Fastening for weights (10 gram)
7. Weights (10 gram).

Assembly of the setup

Before starting the assembly, check if somebody has already done some steps.

Put the trolley on the rails. Check that it does not fall off the rails.

Take the rope, fasten one end of the rope to the trolley and the other to the fastening for weights. Pass the rope over the pulley.

Algorithm of measurements

1. Turn the laptop on.
2. Connect the CASSY unit to the laptop via USB.
3. Run the file **Linear motion** (Desktop\Students\Labs). In the window which will appear, press the button **Close**.
4. Move the trolley on the rail to the right side as far as the rope allows it. Zero the reading of the path s_1 . To do this, press the reading in the s_1 window with the right mouse button and then press **>0<** button in the pop-up menu.
5. Then release the trolley and start the measurements simultaneously. The measurements are started by either pressing **F9** on the keyboard or by the **Measurement...** button on the top of the programme window.

When the trolley reaches the leftmost position, stop the measurements (again, either by the **F9** key or by the **Measurement...** button).

6. Two plots will appear in the window. They may be seen in the **Path** and **Velocity** tabs. They show the time dependencies of the path and speed, respectively.

7. Repeat steps 4, 5 three times.

8. Analyse the obtained data on the speed (**Velocity** tab).

(a) Press the diagram area by the right mouse (or touchpad) button, choose **Approximation** --> **Line** in the appearing menu.

(b) Press the analysed graphic with the left mouse button and drag the cursor (while holding the button) till the end of the linear segment. Thus, you can omit “bad” points in the beginning and in the end of the graphic. Release the button. A straight line passing through the experimental points should appear.

(c) Press the keys **Alt + T**, then press **OK** in the arising window. Move the cursor to the graphic to be analysed and press the left mouse button. An equation should appear nearby.

(d) In this equation the coefficient **A** is the acceleration a with which the trolley had been moving.

9. Save the obtained plots.

10. Write down the acceleration values and the mass of the weight used.

11. Add a new weight to the rope, and repeat steps 4–6.

12. Build a plot for the dependence of the acceleration on the weight mass $a(m)$ (mass on the horizontal axis, acceleration on the vertical axis).

Questions

1. What is the difference between vector and scalar quantities (give examples of vector and scalar in physics).

2. Give definitions: material point, radius vector, path, displacement, speed, velocity, acceleration. What are the units of measurements in SI?

3. What is the difference between average and instantaneous velocity? Write the formulas for both cases.
4. Write the equation and draw plots of the dependence of the path, speed, and acceleration versus time for uniform straight-line motion.
5. Write the equation and draw plots of the dependence of the path, speed and acceleration versus time for straight-line motion with a constant acceleration.
6. How to get the equation of motion if the velocity and acceleration of the body are known at each moment of time?
7. What statement is contained in Newton's first law? What system is called inertial? What is inertia? Give examples to demonstrate the inertia of the bodies.
8. Formulate Newton's second law. Show, using an example from this work, how to write Newton's second law, if several forces act on the body.
9. Formulate Newton's third law. Give examples of forces of action and reaction.
10. Why the obtained plot of acceleration versus the mass of the trolley $a(m)$ does not pass through the origin (0,0)?

113. Forces on an inclined plane

Objective

Studying the forces applied to a body placed on an inclined plane.

Tasks

Applying the Newton's second law to describe the motion of the body on the inclined plane.

Determination of coefficients of static and kinetic friction.

Forces acting on a body on an inclined plane

Gravity force \vec{F}_g is the force of attraction between the body and the Earth. The force is applied to the centre of mass of the body and directed down, towards the earth,

$$\vec{F}_g = m\vec{g} ,$$

where m is mass of the body (kg). In this case, the mass characterises the gravitational interactions. Suppose, one has several particles (parts), with masses m_1 , m_2 , and so on. Let the position vector of m_1 be $\vec{r}_1(x_1, y_1, z_1)$, those of m_2 be $\vec{r}_2(x_2, y_2, z_2)$, and so on. One defines the centre of mass of the system as the point having the position vector given by

$$\vec{r}_{cm} = \frac{m_1 \cdot \vec{r}_1 + m_2 \cdot \vec{r}_2 + m_3 \cdot \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i \cdot \vec{r}_i}{\sum_i m_i} .$$

The value of acceleration due to gravity $|\vec{g}|$ is approximately 9.8 m/s^2 at or near the earth's surface. In fact, the value of acceleration of free fall down varies from point to point on the earth's surface, from 9.78 to 9.82 m/s^2 , because the earth is not perfectly spherical and because of effects due to its rotation and orbital motion. Other factors can influence as well.

Weight of the body \vec{P} is the force which body applies due to gravity or acceleration (including centrifugal force) to the support or suspension, which prevents the body from falling. The force is also directed down.

Reaction force \vec{N} is the force applied from the support to the body. The force is applied perpendicularly to the support's plane towards the body.

When one body is sliding on the surface of another object, kinetic friction force appears which hinders this motion. Friction appears due to the roughness of the surfaces, interaction of microscopic protrusions and bulges on the surfaces. In addition, forces of molecular attraction arise at the points where two objects come into contact with each other.

Friction forces arise pairwise (the first body acts on the second and vice versa) and are oriented antiparallel to the velocity vectors of the relative movement of the bodies (Fig. 113.1.).

According to Newton's third law: $\vec{F}_{fr1} = -\vec{F}_{fr2}$.

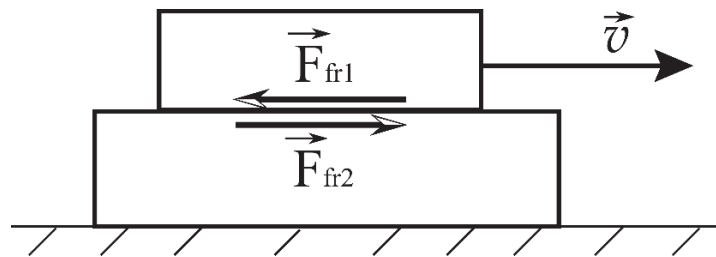


Figure 113.1. Friction forces

The kind of friction that acts when a body slides over a surface is called a kinetic friction force. The value of the kinetic friction force depends on the relative speed of the object relative to the surface: the higher the speed, the less the friction force.

In many cases, the magnitude of the kinetic friction force is found experimentally to be approximately proportional to the magnitude of the normal force:

$$F_{fr} = \mu_k \cdot N , \quad (113.1)$$

where μ_k is the coefficient of kinetic friction (unitless). It is the scalar relation between the magnitudes of the two perpendicular forces, not a vector equation. Sometime, for simplicity, the kinetic friction force can be assumed equal to the static friction force.

Static friction force arises when we try to slide an object across the surface, but object does not move at all $F_{fr} \leq \mu_{stat} \cdot N$, where μ_{stat} is the coefficient of static friction (unitless).

The coefficient of friction depends on the materials from which the bodies are made and on the surface state (rough or polished), and it is specific for each pair of materials. It is found from experiments.

Rolling friction force appears when a round body is rolling on the surface (for example, a wheel on a road). Rolling does not require the bodies to cling to and detach from microscopic roughness, and hence it is much smaller than the kinetic friction force. When the wheel rolls on a surface, it presses in a small dip (Fig. 113.2). Part of the energy is spent to make this deformation, therefore rolling friction is smaller on rigid surfaces (clearly, it is easier to ride a bicycle on a stone road than on the sand).

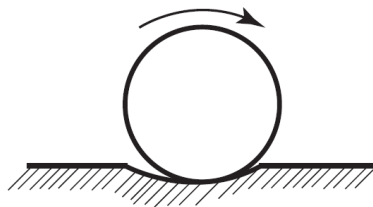


Figure 113.2. Rolling friction force

Task 1. Determining the coefficient of kinetic friction.

A body with the mass m is pulled up at a constant speed across the surface of inclined plane. This is achieved by applying a constant force F parallel to the plane. The plane is lifted to the height h at one of its ends; its basement (projection to the horizontal table) has the length L . The task is to find the coefficient μ_k .

Draw a schematic and mark all forces acting in the system (Fig. 113.3).

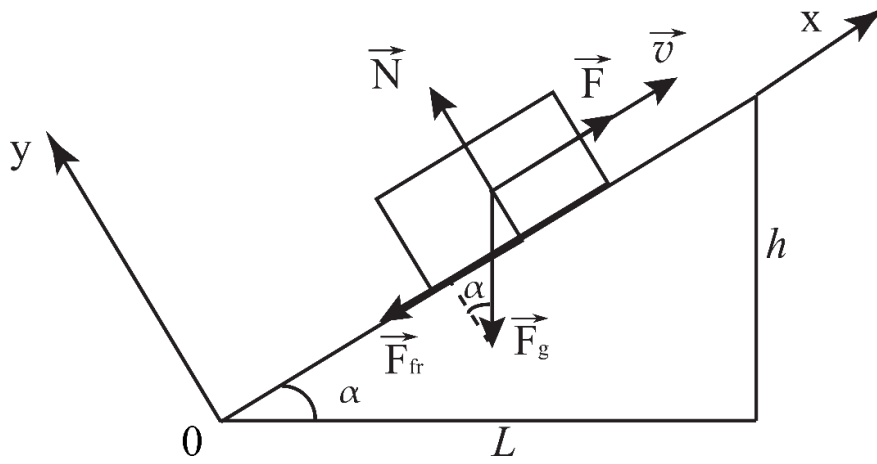


Figure 113.3. Scheme of the experiment

There are four forces acting on the body: friction F_{fr} , normal force N , gravity force mg , and the force F , making the body move (action of the hand). The Newton's second law in this case is written as

$$m\vec{a} = m\vec{g} + \vec{F} + \vec{N} + \vec{F}_{\text{fr}}. \quad (113.2)$$

Let's choose the coordinate system so that the x -axis is parallel to the plane and looks up, and the y -axis is perpendicular to x and oriented as shown in Fig. 113.3. The acceleration $\vec{a} = 0$, so we have straight-line motion with constant velocity. Projection of the forces onto the chosen axes leads to the following system of equations from Eq. (113.2):

$$\begin{cases} 0 = F - F_{\text{fr}} - mg \sin \alpha, & (\text{for } 0x) \\ 0 = N - mg \cos \alpha, & (\text{for } 0y) \end{cases}. \quad (113.3)$$

Friction force can now be expressed as

$$F_{\text{fr}} = F - mg \sin \alpha. \quad (113.4)$$

Eq. (113.4) can then be rewritten using expressions (113.1) and (113.3):

$$F - mg \cdot \sin \alpha = \mu_k \cdot mg \cdot \cos \alpha. \quad (113.5)$$

The coefficient of kinetic friction can be calculated as

$$\mu_k = \frac{F}{mg \cos \alpha} - \tan \alpha. \quad (113.6)$$

Trigonometric expressions can be reformulated using the geometric parameters of the experimental setup h and L :

$$\tan \alpha = h/L \text{ and } \cos \alpha = L/\sqrt{h^2 + L^2} \quad (113.7)$$

Now, combining Eqs. (113.6) and (113.7), we obtain the working formula for calculating the coefficient of kinetic friction μ_k :

$$\mu_k = \frac{F \cdot \sqrt{h^2 + L^2}}{mg \cdot L} - \frac{h}{L}. \quad (113.8)$$

Task 2. Determining the static friction coefficient.

The body is on an inclined plane, is motionless and is influenced by three forces: friction force F_{fr} , normal force N , and the gravity force mg . The static friction force reaches its maximum value when the inclination angle α achieves its maximum, after which the body begins to slide. The coefficient μ_{stat} can be found at this point in time. The base and the height of inclined plate are denoted by L and h , respectively.

The task is solved in the same way as in previous case. Let's draw up Fig. 113.4, all forces and the coordinate system. The motion equation (113.9) is based on Newton's second law, its equivalent form in terms of projections is (113.10).

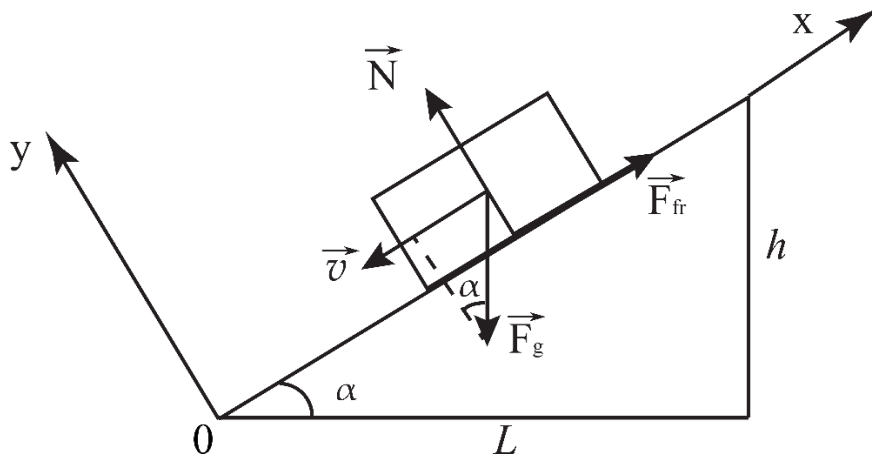


Figure 113.4. To the task of finding the static friction coefficient

$$0 = m\vec{g} + \vec{N} + \vec{F}_{fr}; \quad (113.9)$$

$$\begin{cases} 0 = -F_{fr} - mg \cdot \sin \alpha \text{ (for } 0x), \\ 0 = N - mg \cdot \cos \alpha \text{ (for } 0y) \end{cases} \quad (113.10)$$

The static friction force is calculated as $F_{\text{fr}} = \mu_{\text{stat}} \cdot N$. Substituting that into Eqs. (113.10), we get

$$mg \sin \alpha = \mu_{\text{stat}} mg \cos \alpha . \quad (113.11)$$

Finally

$$\mu_{\text{stat}} = \tan \alpha = h/L . \quad (113.12)$$

Experimental setup

1. Inclined plane
2. Ruler
3. Body (wooden brick with a hook)
4. Dynamometer (spring balance)
5. Movable support (a wooden bar).

Algorithm of measurements

Task 1

1. Determine the weight of the body P . To do this, hang it on the dynamometer and record the reading. This is the weight $P = mg$.
2. Rise the plane on a small angle. This can be done by placing the bar near 35 cm mark. Put the brick with the hook onto the plane.
3. Move the brick upwards along the plane with a dynamometer. Keep it parallel to the plane and try to save constant velocity.
4. Record the dynamometer reading F to the table.
5. Repeat steps 3 and 4 three times. Record h and L also.
6. Repeat steps 2–5 for three different angles.
7. Calculate average force $\langle F \rangle$ for each chosen angle α .
8. Calculate the coefficient of kinetic friction μ_k using Eq. (113.8) and insert it also into table.

Table. Finding of the friction force

	$h, \text{ m}$	$L, \text{ m}$	$F, \text{ N}$	$\langle F \rangle, \text{ N}$	μ_k
1					
2	...				
3					

Task 2

- Put the brick with the hook onto the plane and move the bar slowly to increase the inclination angle. Determine the maximum angle at which the brick begins to slide.
- Repeat step 1 five times. Record the readings h and L to table 2.
- Calculate coefficient of static friction using Eq. (113.12).

Analysis of results

Compare the coefficients of friction obtained in the two tasks.

Questions

- What is a vector?
- What is force?
- Write Newton's second law.
- What is the vector sum of all external forces acting on the body?
- What is the weight of a body? To which objects is the weight force applied, which is its direction?
- Reaction force. What is the reason for the reaction force? How is it related to weight?
- Three forces act on the body: 0.4 N to the right and up at angle of 30° to the horizon, 0.7 N to the right and up at angle of 45° , and 0.8 N to the right and up at angle of 60° to the horizon. Find the projection of the resultant of these forces onto the vertical axis.

8. A body with the mass of 200 g lies on a horizontal table; the coefficient of friction is $\mu = 0.07$. Find the static friction force acting on the body.

9. A body lies on a plane. How does the friction force depend on the angle between the plane and the horizon?

10. Which factors determine the kinetic friction force?

11. Which factors determine the rolling friction force?

12. Imagine a dynamometer that reads 0.04 N at no load. What type of error is it? How should this be resolved?

13. Let 12 consecutive measurements of the same force under the same conditions give the following series of readings: 0.13; 0.12; 0.13; 0.14; 0.12; 0.13; 0.12; 0.14; 0.25; 0.13; 0.14; 0.12 N. Which types of error are here? How should they be considered?

121. Oberbeck's pendulum

Objective

Verification of rotational analogue of Newton's second law for a rigid body using the Oberbeck's pendulum.

Task 1

Rotational motion of a rigid body can be described in terms of the rate of change of angle θ in radians ($360^\circ = 2\pi$ radians). For example, a rotating body makes an angle θ_1 with x -axis at time t_1 and θ_2 at time t_2 . The average *angular velocity* ω in time interval $\Delta t = t_2 - t_1$ is the ratio of the angular displacement $\Delta\theta = \theta_2 - \theta_1$ to Δt : $\omega = \frac{\Delta\theta}{\Delta t}$. In an analogous way to description of straight-line motion, the average *angular acceleration* is $\beta = \frac{\Delta\omega}{\Delta t}$, the usual unit is rad/s^2 . In case of fixed-axis rotation with constant angular acceleration: $\theta = \theta_0 + \omega_0 \cdot t + \frac{\beta \cdot t^2}{2}$.

The *moment of inertia* of the body I is the quantity, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation:

$$I = \sum_i m_i \cdot r_i^2, \text{ SI unit is } \text{kg} \cdot \text{m}^2.$$

Rotational analogue of Newton's second law for a rigid body in the case of a stationary axis is written as:

$$M_z = I_z \beta, \quad (121.1)$$

where M_z is the projection of the resultant moment of force (torque) to the rotation axis, unit is the newton-metre $\text{N} \cdot \text{m}$; I_z is the moment of inertia of the object about this axis; β is the angular acceleration. The quantitative measure of the tendency of a force F to cause or change the rotational motion of a body is torque $M_z = F \cdot l$, the distance l is called the “lever arm”.

If the moment of inertia of the studied system is constant, then a change in the resultant torque causing the motion should lead to a proportional change in the angular acceleration of the system:

$$\frac{M_{z1}}{M_{z2}} = \frac{\beta_1}{\beta_2} \text{ if } I_z = \text{const.} \quad (121.2)$$

A few remarks should be made regarding the movement of the plummet, pulley (about the axis Z_p), and the cross-piece (about the axis Z ; see Fig. 121.1). Friction in the bearing of the cross-piece and the pulley can be neglected in further analysis (the friction coefficient in rolling bearings treated with industrial lubricants is small, ~ 0.001). In our experiment, the mass of the movable pulley, the mass and deformation of the rope (fibre) are also small, so the pulley can be considered weightless, and the rope can be considered weightless and inextensible. This means that the tension is the same along the whole rope's length. Hence, the cross-piece is rotating under the action of the moment of the rope tension force \vec{T} .

Constant forces (the gravity force $m\vec{g}$ and the rope tension force \vec{T}) act on falling plummet m , so the acceleration of the plummet is also constant (no changes with time). Then, we can conclude using the Newton's second law that

$$ma = mg - T \tag{121.3}$$

and

$$T = m(g - a). \tag{121.4}$$

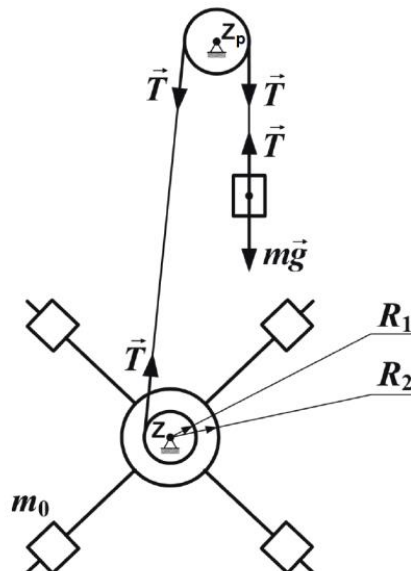


Figure 121.1. Forces in the system

If a body falls with a constant acceleration from the height h during the time t with the zero initial velocity, then its acceleration can be found as

$$a = \frac{2h}{t^2}. \quad (121.5)$$

Therefore, the rope tension force is

$$T = m \left(g - \frac{2h}{t^2} \right). \quad (121.6)$$

Note that if the rope does not slip, the acceleration of the plummet is the same (in absolute value) as the tangential acceleration of the points at the pulley's rim around which the rope is wound. Hence, for the cross-piece rotating with the angular acceleration β we can conclude that $a = \beta R$, where R is the radius of the pulley level on which the rope is wound. Using equation (5), we obtain

$$\beta = \frac{2h}{t^2 R}. \quad (121.7)$$

The projection of the tension force \vec{T} on the axis of rotation Z is defined as follows:

$$M_z = TR = mR \left(g - \frac{2h}{t^2} \right). \quad (121.8)$$

Substituting the parameters calculated using Eqs. (121.7) and (121.8) into the initial Eq. (121.2) we can conclude that the rotational analogue of Newton's second law for a rigid body in the case of a stationary axis is correct.

Task 2

According to Eq. (121.2), a change in the moment of inertia of the cross-piece should lead to a change in the angular acceleration of the system if the resultant moment remains constant, i.e.

$$\frac{I_{z1}}{I_{z2}} = \frac{\beta_2}{\beta_1} \text{ if } M_z = \text{const.} \quad (121.9)$$

Note that in this experiment the work of the friction forces is negligible. The masses of the pulley and the rope are also small, so their mechanical energy can be neglected. Therefore, to determine the moment of inertia of the cross-piece with plummets about the rotation axis Z , one can use the law of conservation of mechanical energy for the system of bodies including the cross-piece with plummets m_0 , the light movable pulley, an inextensible weightless rope, and the plummet m .

Wind the rope around the pulley of the cross-piece to lift the plummet m to the position where the plummet's bottom is at the level of the upper photosensor. Since the distance between the sensors is h , the plummet m receives the potential energy $U = mgh$ relative to the lower sensor's level. If the plummet is released, it will move downward with the acceleration a , and the cross-piece will rotate with the angular acceleration β . The plummet's (m) potential energy U will be converted into the kinetic energy of translational motion $mv^2/2$ and into the kinetic energy of rotation of the cross-piece with plummets m_0 and the two-level pulley $I_z\omega^2/2$ (ω is the angular velocity). The law of conservation of energy requires that

$$mgh = \frac{mv^2}{2} + \frac{I_z\omega^2}{2}, \quad (121.10)$$

where the falling plummet's speed v and the angular velocity ω correspond to the time moment when the plummet m is at the level of the lower photosensor.

From Eq. (121.5) and knowing that $v = at$ we get the proportion

$$v = \frac{2h}{t}. \quad (121.11)$$

The angular velocity of the cross-piece ω is related to the plummets speed of falling as

$$\omega = \frac{v}{R} = \frac{2h}{Rt}, \quad (121.12)$$

where R is the radius of the pulley's level on which the rope is wound.

Combining the Eqs. (121.11), (121.12), and (121.10), we can get the expression for determination of the moment of inertia of the cross-piece with the plummets and the pulley:

$$I = mR^2 \left(\frac{gt^2}{2h} - 1 \right). \quad (121.13)$$

Substituting the values found from Eqs. (121.7) and (121.13) into Eq. (121.9) we can see that the rotational analogue of Newton's second law for a rigid body in the case of a stationary axis is correct.

Experimental setup

1. Cruciform Oberbeck's pendulum
2. Electronic unit with photoelectric sensors
3. Rope with plummet
4. Ruler
5. Callipers

The setup for studying rotational motion about a stationary axis is shown in Fig. 121.2. The column 1 is attached to the base 2; the pendulum, lower fixed support 5 and upper movable support 6, lower and upper sleeves 7 and 8 are attached to the column 1. The pendulum consists of the cross-piece 3 with detachable plummets 4 having the mass of m_0 and the two-level pulley 11, which can freely rotate around the stationary horizontal axis. A movable light block 9 is attached to the upper sleeve 8. The rope 10 passes through the block 9; one end of the rope is fixed on the two-level pulley 11 having the radii of levels R_1 and R_2 .

At the other end of the rope the plummet 12 of the mass m is fixed. The lower sleeve also holds a board 13 with an electromagnet 14 which is used to fix the cross-piece by a friction clutch. Both supports hold photoelectric sensors 15 separated by a distance h which can be changed by moving the support 6 along the column. This distance is measured by a scale drawn on the column. On the moment when the plummet 12 passes through the upper sensor, the timer begins measuring the time. The measurement ends when the plummet passes through the lower photosensor; brake electromagnet 14 is activated at the same moment. Thus, the time t necessary for the plummet 12 to go through the distance h between the photosensors 15 is measured.

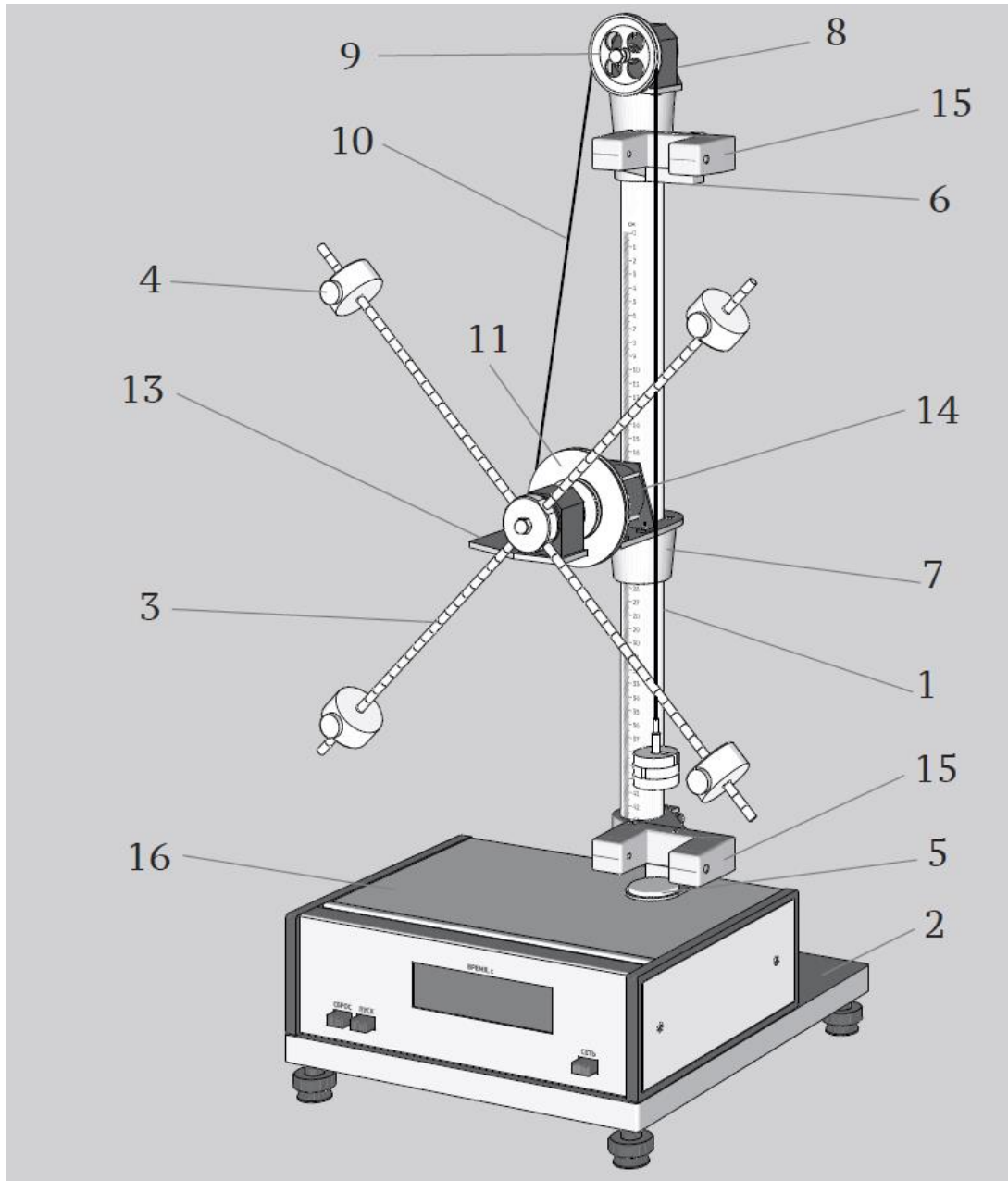


Figure 121.2. Experimental setup

Algorithm of measurements

1. Switch up the setup by pressing the **Сеть/Power** button. Lamps of the photosensors and the timer display should start to light up.
2. Press the **Нулк/Start** button and wind the rope around the pullet of the cross-piece to lift the plummet m to the position where the plummet's bottom is at the level of the upper photosensor.

The plummet must not cross the light beam in the sensor. In this position the plummet has the height h relative to the lower sensor.

3. Push up the **Пуск/Start** button; the electromagnet of the clutch should switch on and hold the cross-piece in its initial position.

4. Press the **Сброс/Clear** button to prepare the timer.

5. Press the **Пуск/Start** button. The magnet is switched off releasing the cross-piece, and the plummet m begins to move downward. The duration of the descent is measured by the electronic timer, which switches on and off when the plummet crosses the photosensors' light beams.

Algorithm of measurements for task 1

1. Fasten plummets m_0 on the rods of the cross-piece at equal distances r from the axis of rotation (choose the value of r yourself).

2. Measure the radii of the pulley's levels R_1 and R_2 using callipers.

3. Measure the height h (the distance between the sensors 15) by scale on the column 1.

4. Fasten the rope with the plummet m (the mass m is reported by the teacher) on the pulley's level R_1 . Release the plummet and measure the falling time t_1 from height h . Repeat the measurements 3-5 times and find the average value $\langle t_1 \rangle$.

5. Repeat step 4 with R_2 and find $\langle t_2 \rangle$.

6. Substitute obtained values into Eq. (121.7) to calculate the angular accelerations of the cross-piece β_1 and β_2 , and in Eq. (121.8) to calculate the projections of the resultant moments of forces M_{z1} and M_{z2} .

7. To check the rotational analogue of Newton's second law for a rigid body in the case of a stationary axis, substitute the values β_1 and β_2 , M_{z1} and M_{z2} into the proportion (121.2).

8. Estimate the accuracy of one of the obtained parameters.

9. Draw your conclusions.

Algorithm of measurements for task 2

1. Fasten plummets m_0 on the rods of the cross-piece at equal distances r_1 from the axis of rotation (the value of r_1 is reported by the teacher).
2. Measure the radii of the pulley's levels R_1 and R_2 using calipers (use the same level in all further measurements!).
3. Measure the height h (the distance between the sensors 15) by scale on the column 1.
4. Fasten the rope with the plummet m (the mass m is reported by the teacher) on the pulley's level R_1 or R_2 , depending on your choice. Release the plummet and measure the falling time t_1 from height h . Repeat the measurements 3-5 times and find the average value $\langle t_1 \rangle$.
5. Move the plummets m_0 on the rods to a new position r_2 (at the choice of the teacher).
6. Measure the time t_2 3-5 times as in step 4 and find the average value $\langle t_2 \rangle$.
7. Substitute obtained values into Eq. (121.7) to calculate the angular accelerations of the cross-piece β_1 and β_2 , and in Eq. (121.13) to calculate the moments of inertia I_{z1} and I_{z2} .
8. To check the rotational analogue of Newton's second law for a rigid body in the case of a stationary axis, substitute the values β_1 and β_2 , I_{z1} and I_{z2} into the proportion (121.9).
9. Estimate the accuracy of one of the obtained parameters.
10. Draw your conclusions.

Questions

1. Definition of an absolutely rigid body.
2. What is the difference between translational and rotational motion of a rigid body? Give some examples.
3. Definition of the rotation axis of a rigid body. Definition of the angular velocity. What is the direction of the angular velocity vector?

4. Relationship between linear and angular velocities.
5. How many times the angular velocity of the minute hand of a mechanical watch is greater than the angular velocity of the hour hand?
6. Definition of a torque. Draw a figure with an explanation.
7. Definition of the angular acceleration.
8. Definition of the moment of inertia of the body. Write the formula and units of measure.
9. Why, speaking about the magnitude of the moment of inertia of the body, it is necessary to indicate, about which axis it is determined?
10. Rotational analogue of Newton's second law for a rigid body.

122. Moment of inertia of a flywheel

Objective

Studying the parallel axis theorem (Huygens–Steiner theorem).

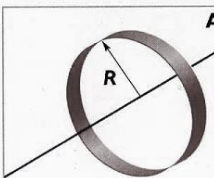
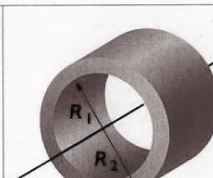
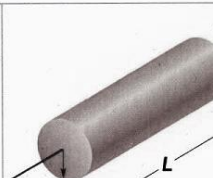
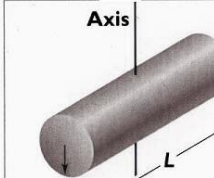
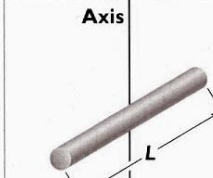
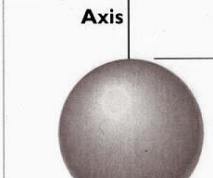
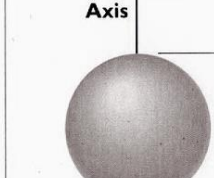
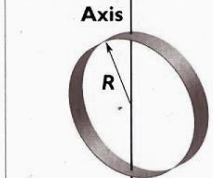
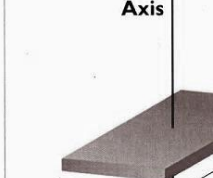
Tasks

Acquaintance with the method of measuring the moment of inertia of a body using oscillations.

Determination of the moment of inertia of a flywheel.

The moment of inertia of the body I is the quantity, obtained by multiplying the mass of each particle by the square of its distance from the axis of rotation:

$$I = \sum_i m_i \cdot r_i^2, \text{ SI unit is } \text{kg} \cdot \text{m}^2.$$

 <p>Axis</p> <p>Hoop about central axis</p> <p>$I = MR^2$</p> <p>(a)</p>	 <p>Axis</p> <p>Annular cylinder (or ring) about central axis</p> <p>$I = \frac{1}{2}M(R_1^2 + R_2^2)$</p> <p>(b)</p>	 <p>Axis</p> <p>Solid cylinder (or disk) about central axis</p> <p>$I = \frac{1}{2}MR^2$</p> <p>(c)</p>
 <p>Axis</p> <p>Solid cylinder (or disk) about central diameter</p> <p>$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$</p> <p>(d)</p>	 <p>Axis</p> <p>Thin rod about axis through center perpendicular to length</p> <p>$I = \frac{1}{12}ML^2$</p> <p>(e)</p>	 <p>Axis</p> <p>Solid sphere about any diameter</p> <p>$I = \frac{2}{5}MR^2$</p> <p>(f)</p>
 <p>Axis</p> <p>Thin spherical shell about any diameter</p> <p>$I = \frac{2}{3}MR^2$</p> <p>(g)</p>	 <p>Axis</p> <p>Hoop about any diameter</p> <p>$I = \frac{1}{2}MR^2$</p> <p>(h)</p>	 <p>Axis</p> <p>Slab about perpendicular axis through center</p> <p>$I = \frac{1}{12}M(a^2 + b^2)$</p> <p>(i)</p>

From <https://sites.google.com/site/mrloboscience/>

There is a simple relationship between the moment of inertia I_{cm} of a body of mass m about an axis through its centre of mass and the moment of inertia I_p about any other axis parallel to the original one but displaced from it by a distance d : $I_p = I_{\text{cm}} + m \cdot d^2$ (the parallel axis theorem).

Experimental setup

1. Flywheel on a mount
2. Additional body
3. Timer
4. Callipers
5. Balance

The experimental setup (Fig. 122.1 (left)) is a massive flywheel which can rotate about a horizontal axis with almost without friction. The axis passes through the centre of mass of the wheel, so the wheel is in the state of indifference. If an additional body is fixed on the wheel rim, then a position of stable equilibrium appears. If the system is taken out of this equilibrium position by rotating it at an angle α_m , and then released, the wheel will begin to oscillate with a period T . If the angle α_m is small, the oscillation can be considered as harmonic oscillation following the formula $\alpha = \alpha_m \sin(\omega_0 t)$.

When the perturbed system passes through the equilibrium position, its angular velocity reaches the maximum value $\alpha_m \cdot \omega_0$, so the maximum kinetic energy is

$$E_m = \frac{I \alpha_m^2 \omega_0^2}{2}.$$

The moment of inertia I of the system includes the moment of inertia I_w of the flywheel itself and of the additional body I_b .

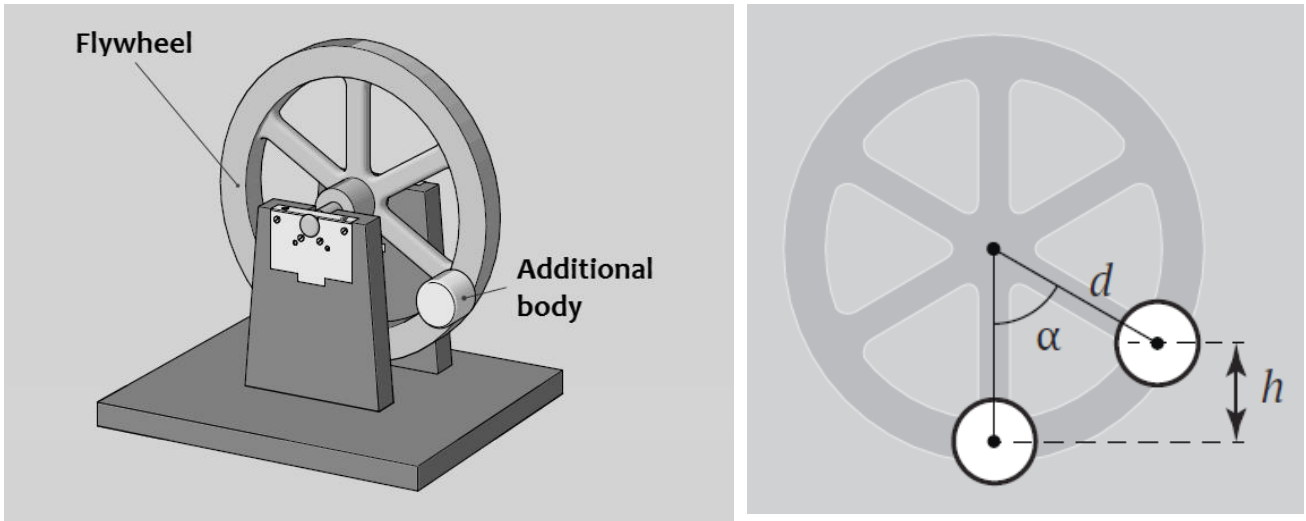


Figure 122.1. Setup

On the other hand, the potential energy of the system is $E = mgh$, where m is the mass of the additional body, and h is the height to which it is raised from the equilibrium position. Geometrical consideration (Fig. 122.1 (right)) shows that

$$h = d - d \cos \alpha = 2d \sin^2 \frac{\alpha}{2},$$

where d is the distance from the centre of the wheel to the centre of mass of the additional body.

In the case of small-amplitude oscillations (only such oscillations can be considered as harmonic in our case) we can use the approximation $\sin \alpha \sim \alpha$. If friction can be neglected, we can assume that the maximum values of potential and kinetic energies are equal (based on the law of conservation of energy). If ω_0 is rewritten using the period, we get the following expression for the moment of inertia of the flywheel:

$$I_w = I - I_b = mgd \frac{T^2}{4\pi^2} - I_b. \quad (122.1)$$

The values on the right side of Eq. (122.1) can be measured directly (g is gravity acceleration or acceleration of free fall), and the value of I_b can be found using the parallel axis theorem:

$$I_b = I_0 + md^2 . \quad (122.2)$$

Here, I_0 is the moment of inertia of the additional body about the axis through the body's centre of mass and parallel to the system's (wheel's) axis of rotation. In this work, the cylindrical body used with the moment of inertia:

$$I_0 = mR^2 / 2 , \quad (122.3)$$

where R is the radius of the cylinder.

Algorithm of measurements

1. Unscrew the additional body and measure its mass.
2. Find the diameter $2R$ of the body and the distance d (Fig. 122.1).
3. Using Eqs. (122.2) and (122.3), calculate the moments of inertia of the additional body about its axis of symmetry (I_0) and the axis of rotation of the wheel (I_b).
4. Attach the body to the wheel rim.
5. Rotate the wheel a little and release it. The wheel will oscillate.
6. Find time t corresponding to as many oscillations as possible (N), but no less than 10. Find the average period of one oscillation: $T = t/N$.
7. Repeat steps 5–6 five times and find the average of the period $\langle T \rangle$.
8. Calculate the moment of inertia of the wheel using Eq. (122.1). Estimate the accuracy of the experiment.

Questions

1. Write down the equation of motion of the flywheel with an additional body on the rim, estimate at what angles the solution of this equation can be considered a harmonic function within the level of accuracy of the instruments in this work.
2. Derive Eq. (122.1).

3. How does the friction force affect the accuracy of measurements? Which features of the experimental setup allow neglecting the friction force?

4. How do the moment of inertia and mass of the additional body affect the accuracy of measurements? What requirements should meet the body?

5. Compare the method described here with other methods of measuring the moment of inertia that you know.

6. Draw plot dependencies of the angular coordinate, angular velocity, angular acceleration of the wheel with the additional body versus time.

131. Torsional pendulum

Objective

Studying the laws of conservation in dynamics of rotational motion.

Task

Finding the speed of a bullet.

Let's consider a particle of constant mass m . The product of the particle's mass and velocity \vec{v} is the momentum: $\vec{p} = m \cdot \vec{v}$. SI unit is kg·m/s.

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant. This is simplest form of the principle of conservation of momentum.

If in isolated system the forces between bodies are conservative, so the total kinetic energy of the system is the same after the collision as before. Such a collision is called a completely elastic collision. A collision in which the total kinetic energy after the collision is less than before, the collision is called an inelastic collision. A collision in which the colliding bodies stick together and move as one body after the collision is called a completely inelastic collision.

Experimental setup

This work is performed on a rotational ballistic (torsional) pendulum (Fig. 131.1). Base 1 stands on levelling legs 2. The base holds the column 3 with fixed arms 4, 5, 6. The middle arm 5 holds a shooting device 7, a transparent screen 8 with a goniometer, and a photoelectric sensor 9. Arms 4 and 6 have clamps for attaching a steel wire 10. The pendulum hangs on this wire and consists of two rods 11, plasticine covered plates 12, and two movable plummets 13. Photoelectric sensor is connected to a timer 14 which is used for measuring the oscillation period of the pendulum.

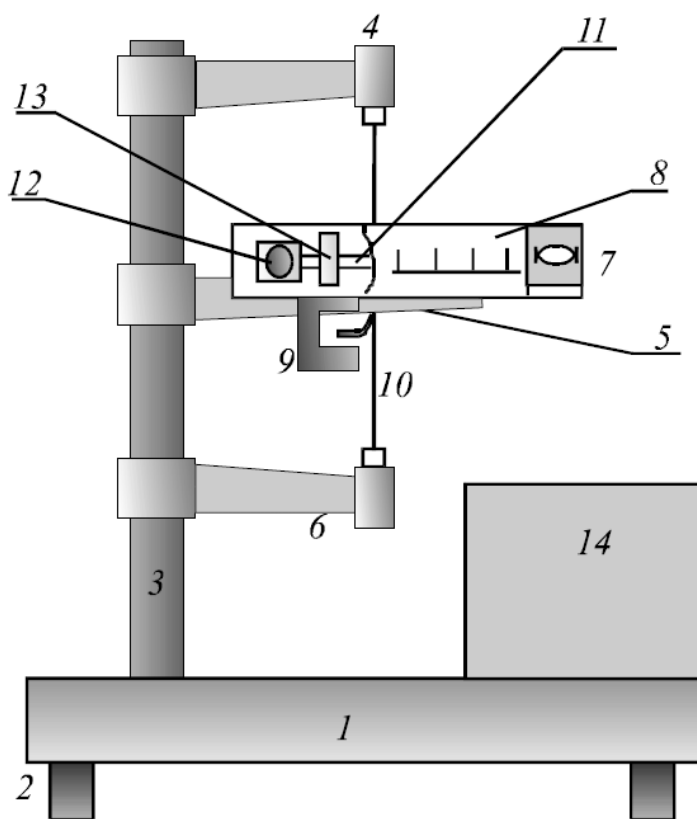


Figure 131.1 Setup

After the bullet hits the pendulum, it is beginning to oscillate about the vertical axis. If the interaction between the bullet and the pendulum is fast enough (the time of interaction is much shorter than the period of oscillation), then the angular momentum should remain the same before and after the shot:

$$mvl = (I_1 + ml^2)\omega, \quad (131.1)$$

where m is the bullet's mass, v is its speed, l is the distance from the rotation axis to the point at which the bullet hits, I_1 is the moment of inertia of the pendulum about the rotation axis, and ω is the angular velocity that the pendulum acquires after the shot.

If friction is neglected, the mechanical energy should conserve during the oscillations. Then the maximum value of the kinetic energy is equal to the maximum value of the potential energy:

$$(I_1 + ml^2)\omega^2/2 = D\alpha_m^2/2. \quad (131.2)$$

Here D is the torsion constant (the proportionality factor in the ratio between the moment of tension forces (torque) and the angle of rotation, similar to the stiffness of a spring in the Hooke's law), and α_m is the angle of the maximum deviation of the pendulum.

Eqs. (131.1) and (131.2) allow finding the following expression for the bullet's speed:

$$v = \frac{\alpha_m}{ml} \sqrt{D(I_1 + ml^2)}. \quad (131.3)$$

Since in our experiment $ml^2 \ll I_1$, we can simplify this formula and write down:

$$v = \frac{\alpha_m}{ml} \cdot \sqrt{D \cdot I_1}. \quad (131.4)$$

To find I_1 and D , free oscillations of the pendulum can be used. The motion of equation is written using the derivative of the unknown function of time $\alpha(t)$:

$$I_1 \alpha'' = -D\alpha. \quad (131.5)$$

The solution of this differential equation is the harmonic function $\alpha = \alpha_m \cos(2\pi t/T + \varphi_0)$ with the period T depending on the properties of the system as:

$$T = 2\pi \sqrt{\frac{I_1}{D}}. \quad (131.6)$$

If the distance between plummets 13 is changed (Fig. 131.1), the moment of inertia of the pendulum also changes, and hence, the period of oscillations. For two different positions of plummets, we can write periods:

$$T_1 = 2\pi \sqrt{\frac{I_1}{D}}, \quad T_2 = 2\pi \sqrt{\frac{I_2}{D}}, \quad (131.7)$$

moments of inertia are

$$I_1 = I_0 + 2MR_1^2, \quad I_2 = I_0 + 2MR_2^2. \quad (131.8)$$

Here I_0 is the moment of inertia of the pendulum itself without the plummets; R_1 and R_2 are the distances from the rotation axis to the plummets' centres, and M is the mass of the plummet. The values of I_1 and D can be calculated from these equations, but we do not need to know the numerical values. If corresponding analytical expressions substitute

into Eq. (131.3), the working formula for finding the bullet's speed can be written as:

$$v = \frac{4\pi\alpha_m M}{ml} \frac{T_1}{T_1^2 - T_2^2} (R_1^2 - R_2^2). \quad (131.9)$$

Algorithm of measurements

1. Switch on the setup by pressing the **Сеть/Power** and **Сброс/Clear** buttons in subsequence on the front panel. Zeros should start to glow on the digital display. Check if the lamp in the photoelectric sensor is also working.

2. Place the plummets 13 on the rods at the maximum distance from each other, measure R_1 .

3. Set the angle in the zero position, if necessary, by rotating the element holding the wire in the arms 4 and 6.

4. Insert the bullet into the shooting device, turn the clamp, compress the spring, and make a shot by turning the handle.

5. Measure the maximum angle of rotation of the pendulum.

6. Skip two or three first oscillation and measure the duration of the next 10–15 periods by starting the timer with the **Сброс/Clear** button and stopping it with the **Стоп/Stop** button. The number of oscillations counted is shown on the display **Периоды/Periods**. Calculate T_1 .

7. Place the plummets 13 at the minimum distance R_2 ; measure it.

8. Rotate the pendulum by hand (manually) at an angle equal to α_m and release it. Find T_2 like in the step 6.

9. Measure the bullet's mass m .

10. Calculate the bullet's speed from Eq. (131.9). The mass of the plummet is indicated on it.

Questions

1. Mechanics of collision (impact), basic features of this phenomenon.

2. Completely elastic and completely inelastic collisions: common features and differences.

3. Examples of completely inelastic collisions.
4. Describe the experimental setup and the method of measuring the bullet's speed.
5. Is it possible to measure any speed of a bullet? Is it possible to deal with a bullet of any mass?

132. Maxwell's wheel

Objective

Calculating and measuring the moment of inertia of a cylindrical solid about the axis of symmetry.

Tasks. Moments of inertia determination of the Maxwell's wheel and rings.

Energy is the quantitative property that must be transferred to a body or physical system to perform work on the body, or to heat it. The energy is possibility for work. SI unit is joule J. Energy is never created or destroyed; it changes form only. The kinetic energy: $K = \frac{m \cdot v^2}{2}$, rotational kinetic energy: $K = \frac{I \cdot \omega^2}{2}$, gravitational potential energy $U = m \cdot g \cdot h$, elastic potential energy $U = \frac{k \cdot x^2}{2}$, m is body mass, v is speed, I is moment of inertia, ω is the magnitude of the body's angular velocity, g is acceleration of free fall down 9.8 m/s^2 , h is height, k is the force constant of the spring, x is distance of stretching.

Moment of inertia of a material point of the mass m about some rotation axis is the value $I = mr^2$, where r is the distance between the point and the axis. A rigid body can be considered as the system of material points having the masses m_i and located at the distances r_i from the axis, so its moment of inertia will be

$$I = \sum_i m_i r_i^2. \quad (132.1)$$

In the case of continuous distribution of mass, the expression for I takes the following form:

$$I = \int r^2 dm. \quad (132.2)$$

The limits of integration should include the whole body.

It can be calculated that for a cylinder having the radius R and mass m , the moment of inertia about its axis of symmetry is:

$$I = \frac{1}{2} m R^2, \quad (132.3a)$$

and for the cylinder having a coaxial cavity drilled out along its axis (in other words, a thick-walled cylindrical tube with open ends having the inner radius R_1 and the outer radius R_2), the moment of inertia is

$$I = \frac{1}{2}m(R_1^2 + R_2^2). \quad (132.3b)$$

Maxwell's wheel is a massive disk with the axis suspended on two ropes that can wound about the axis (Fig. 132.1 (left)). If you wind the rope, the wheel goes upward; if the wheel is released, it performs reciprocating motion in the vertical plane, while the wheel rotates about its axis at the same time.

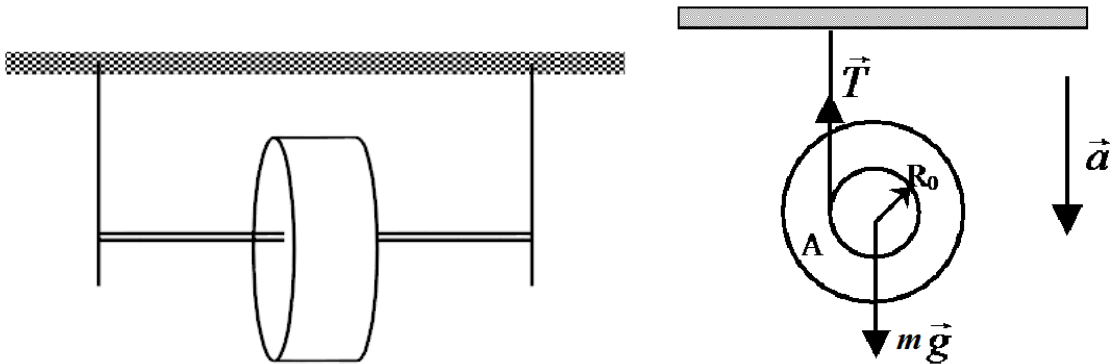


Figure 132.1. Maxwell's wheel

The forces acting in the system are shown in Fig. 132.1 (right). To analyse the motion of the wheel, it is convenient to choose the frame of reference with the centre at the centre of mass A of the wheel. The centre of mass moves down with a linear acceleration \vec{a} . The equation of motion of the wheel's centre of mass is written as

$$m \cdot \vec{a} = m \cdot \vec{g} + \vec{T}, \quad (132.4)$$

where \vec{T} is the resulting tension force of both ropes, and m is the wheel's mass.

In addition, the wheel performs rotational motion about the horizontal axis (through the centre of mass) due to the moment of the rope tension force: $M = R_0 T$, where M is the moment of the force \vec{T} , and R_0 is the moment arm (radius of the rod).

Rotational analogue of Newton's second law for a rigid body in the case of a stationary axis is written as:

$$\vec{M} = \vec{I} \times \vec{\beta}, \quad (132.5)$$

where \vec{M} is the moment of force (torque) to the rotation axis; \vec{I} is the moment of inertia of the object about this axis; $\vec{\beta}$ is the angular acceleration. Projection on the x -axis $M_x = I_x \beta$. We can write the expressions for scalars now:

$$\begin{cases} ma = mg - T & (132.6) \\ M_x = I_x \beta. & (132.7) \end{cases}$$

The distance x that the centre goes down is equal to the length of the rope unwound from the rod:

$$x = \varphi R_0, \quad (132.8)$$

where φ is the total angle of rotation. Differentiating this expression twice with respect to time, we get

$$a = \frac{d^2 x}{dt^2} = R_0 \frac{d^2 \varphi}{dt^2} = R_0 \beta, \quad (132.9)$$

and taking Eq. (132.9) into account, we can rewrite Eq. (132.7) as

$$R_0 T = I_x \cdot \frac{a}{R_0} \text{ or } T = I_x \cdot \frac{a}{R_0^2}. \quad (132.10)$$

Combining Eqs. (132.6) and (132.10), we get

$$a = \frac{mg}{m + I_x / R_0^2}, \quad (132.11)$$

$$T = \frac{mg}{1 + \frac{mR_0^2}{I_x}}. \quad (132.12)$$

Eqs. (132.11) and (132.12) showed that acceleration and the rope tension force are constant (independent of time). Therefore, if the wheel coordinate is determined from the upper attachment point, then time dependence of the coordinate will be

$$x = at^2/2. \quad (132.13)$$

Substituting (132.13) into (132.11), we obtain the following expression for the moment of inertia of the Maxwell's wheel:

$$I_x = mR_0^2 \left(\frac{gt^2}{2x} - 1 \right) \quad (132.14)$$

or

$$I_x = \frac{mD_0^2}{4} \left(\frac{gt^2}{2x} - 1 \right), \quad (132.15)$$

which contains parameters that are easy to measure. R_0 (D_0) is the outer radius (diameter) of the rod together with the rope wound on (about) it, t is the time required for the wheel to go down by the distance x , and m is the wheel's mass. The mass is summed up from the mass of the rod m_0 , the disk m_d , and a ring m_r which, if desired, could be put on the disk.

Experimental setup

The base (Fig. 132.2) stands on levelling legs. The base holds the column 1 with fixed upper arm 2 and movable lower arm. The upper arm holds the electromagnet, photoelectric sensor 3, and the knob for fixing and regulating the length of the rope 4. The lower arm with the photosensor 5 can be moved and fixed in different positions on the column.

The wheel 6 is set on the cylindrical rod 7; rings (thick-walled cylindrical tubes with open ends) 8 can be put on the wheel to change the moment of inertia of the system.

The wheel (with a ring) is kept in the upper position by the electromagnet. The column has a millimetre scale to measure the displacement x of the wheel. Photoelectric sensors are connected to the timer 9.

The front panel of the timer is shown in Fig. 132.2. The **Сеть/Power** button switches the setup on; the lamps in the photosensors should start to glow. The **Сброс/Clear** button is used to set the timer to zero. The **Пуск/Start** button controls the electromagnet: when this button is pressed, the magnet is turned off and the timer goes to the time measurement mode.

Algorithm of measurements

The lower arm of the setup should be set to the lowest position.

1. Put one of the rings on the wheel by pressing it on the side as much as possible.

2. Choose the rope length so that the edge of the metal ring in the lowest position is two millimetres below the photoelectric sensor. At the same time, correct the position of the wheel so that its axis is parallel to the base (horizontal). Adjustment is made using the knob 8 (Fig. 132.2).

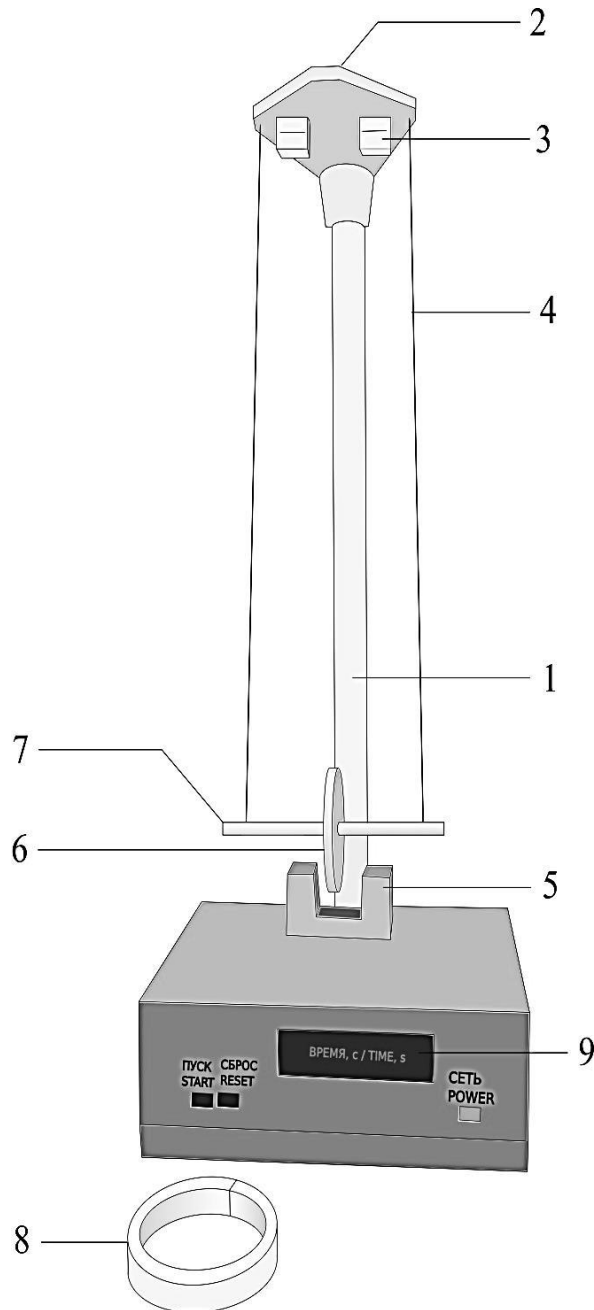


Figure 132.2 Setup

3. Press the **Сеть/Power** button.
4. Wind the rope about the rod coil by coil. Check the winding for uniformity.
5. Fix the wheel at the magnet and rotate it at a small angle ($\sim 5^\circ$).
6. Press the **Сброс/Clear** button.
7. Press the **Пуск/Start** button.
8. Insert the measured value of fall time t into the table.
9. Take 10 measurements of fall time and calculate the average value of t_{av} .
10. Find the distance x using the scale on the column.
11. Find the diameter of the rope D_r and the wheel's rod in several different directions transverse to axis D ; calculate the average values. Calculate $D_0 = D + D_r$ ($R_0 = D_0/2$).
12. Find the wheel's mass, including the mass of the ring (all necessary data are indicated on the parts used).
13. Next steps depend on the task.

Task 1. Moments of inertia determination of the Maxwell's wheel

1. Calculate the moment of inertia theoretically I_{theor} using formulas (132.3a) and (132.3b). The final result is the sum of the moments of inertia of the rod I_r , the disk I_d , and the additional ring I_{ring} . Abovementioned formulas show that $I_r = \frac{1}{2}m_r R_0^2$, where m_r and R_0 are the mass and radius of the rod, $I_d = \frac{1}{2}m_d(R_1^2 + R_0^2)$, where R_1 is the outer radius of the disk, and $I_{ring} = \frac{1}{2}m_{ring}(R_1^2 + R_2^2)$, where R_2 is the outer radius of the ring.
2. Find the moment of inertia experimentally I_{exp} using Eq. (132.14).
3. Analyse the results.

Task 2. Moments of inertia determination of the rings

1. Calculate the moment of inertia theoretically I_{theor} using Eq. (132.3b). The mass m_{ring} is indicated on the ring; R_1 and R_2 are the inner and outer radii of the ring, $I_{\text{ring}} = \frac{1}{2}m_{\text{ring}}(R_1^2 + R_2^2)$.
2. Determine experimentally the moment of inertia I_1 of the Maxwell's wheel without the ring.
3. Determine experimentally the moment of inertia I_2 of the Maxwell's wheel with rings.
4. Find the moment of inertia of the rings I_{exp} .
5. Analyse the results, i.e. build plot I_{exp} vs. I_{theor} for rings.

Table

№	m_r , kg	m_d , kg	m_{ring} , kg	R_0 , m	R_1 , m	R_2 , m	x , m	t , s	t_{av} , s
...									

Questions

1. The theorem of the motion of the mass centre of a system of material points.
2. Definition of the moment of inertia of a material point and a system of material points.
3. Equation of motion of the Maxwell's wheel.
4. Dependence (behaviour) of the acceleration, velocity, and the rope tension force during the wheel's motion.
5. Dependence (behaviour) of the mechanical energy of the Maxwell's wheel during the motion.

141. Simple pendulum

Objective

Measurement of acceleration of free fall (gravity acceleration).

Tasks

Studying the method of measuring of the acceleration of free fall with the help of a simple pendulum.

Estimation of possibility of describing the given real pendulum by the model of a simple pendulum.

Acceleration of free fall \vec{g} is the acceleration with respect to the Earth at which a released body begins to fall down. This acceleration is defined by the sum of the force of gravity (attraction to the Earth) and the centrifugal force of inertia.

A simple (mathematical) pendulum is an imaginary pendulum with all its mass located at one point, while the distance l from this point to the centre of suspension (pivot) being constant during oscillations. Simple calculations show that at small angles of deviation from the vertical, the period of oscillations of the pendulum is:

$$T = 2\pi \sqrt{\frac{l}{g}} . \quad (141.1)$$

Now, the idea of one possible approach to determining the acceleration of free fall is clear: it is necessary to measure the length and period of a simple pendulum.

However, a question arises whether properties of a real pendulum are properly described by the model of a simple pendulum?

Note that Eq. (141.1) shows that the period of oscillations of the simple pendulum is proportional to $l^{0.5}$. If this correlation is true for a given real pendulum, it can be considered a simple pendulum, and the acceleration of free fall will be defined from the formula:

$$g = \frac{4\pi^2 l}{T^2} . \quad (141.2)$$

Experimental setup

1. Massive ball on inextensible rope
2. Ruler
3. Timer

Algorithm of measurements

1. Shorten the rope so that its length is 10-15 cm.
2. Measure the length of the pendulum l , which is the distance between the centre of suspension (pivot) and the centre of the ball.
3. Deflect the ball so that the angle between the rope and the vertical does not exceed 10° , and release the ball.
4. Measure the duration of 10 full oscillations t_{10} and find the period $T = t_{10}/10$.
5. Add 5-10 cm to the length of the pendulum (use a roller at the attachment point).
6. Repeat steps 2-4.
7. Repeat steps 5-6 until the pendulum is over 100 cm long.
8. Analyse the results.

Fill in the table with results of your measurements and calculations.

N	l , cm	t_{10} , s	T , s	T^2 , s ²
1				
2				
...				

Plot the graph T^2 versus l . Do a linear approximation, find slope (acceleration of free fall) and intercept (141.2). Carry out a standard statistical analysis of this plot: calculate average value $\langle g \rangle$, dispersion, and error of determination of the acceleration of free fall.

Questions

1. Forces of inertia.
2. Equations of motion of a material point relative to a rotating frame (the Earth).
3. The reasons for the dependency of the acceleration of free fall on the position on the Earth's surface.
4. Vector and vector's components; vector coordinates; projection of a vector onto a given direction.
5. Angular velocity and angular acceleration.
6. Inertial and non-inertial coordinate system.
7. Moment of inertia.
8. Centre of masses.
9. Acceleration of free fall.

142. Kater's pendulum

Objective

Measurement of acceleration of free fall.

Tasks

Studying the method of measuring of the acceleration of free fall with the help of a Kater's pendulum.

Acceleration of free fall \vec{g} is the acceleration with respect to the Earth with which a body begins to fall. This acceleration is determined by the sum of gravitational interaction with the Earth and the centrifugal inertia force.

The value of g can be found using a physical pendulum. A physical pendulum is an absolutely rigid body that can swing (oscillate) about a stationary horizontal axis. If there are no friction forces, the pendulum's equation of motion looks like:

$$I \frac{d^2\phi}{dt^2} = -mgd \sin \phi, \quad (142.1)$$

where m is the mass of the body, I is its moment of inertia relative to the point of suspension, d is the distance from the point of suspension (pivot) to the centre of mass of the pendulum, and ϕ is the angle by which the pendulum deviates from the equilibrium position. In the case of small oscillations, we can replace $\sin\phi$ by ϕ in the above eq. This gives the equation for harmonic oscillations having the period:

$$T = 2\pi \sqrt{\frac{I}{mgd}}. \quad (142.2)$$

A special case of a physical pendulum is the so-called simple pendulum. This is an idealized pendulum, the total mass of which is concentrated at one point. In this case, we can simplify Eq. (142.2) since $I = ml^2$, and $d = l$ (length of the pendulum), and finally get for g :

$$g = \frac{4\pi^2 l}{T^2}. \quad (142.3)$$

Thus, an idea how to measure the acceleration of free fall is clear: the length and the period of oscillations of a simple pendulum should be measured.

Comparing Eqs. (142.2) and (142.3) we can see that a physical pendulum oscillates in the same way (with the same period) as a simple pendulum having the length of $l = I/(m \cdot d)$. This value is called the reduced pendulum length. The point lying on a straight line passing through the centre of mass at the distance l from the point of suspension is called the centre of oscillation. If a pendulum is suspended at its centre of oscillation, then the period of oscillations will not change (this statement is called parallel axis theorem, or Huygens–Steiner theorem).

It is important to note that the same period can also be achieved, when the pendulum is suspended at any of the points from a certain infinite set of points. According to this definition, the point of suspension and centre of oscillations are the respective points, but they are not the only possible pair of such points. That is why the distance between respective points (which is easy to find by checking the equality of the periods) is not always the reduced length. The distance between respective points is equal to the reduced pendulum length only if these points lie on the same line with the centre of mass and on opposite sides of the centre of mass.

Experimental setup

Kater's (reversible) pendulum consists of a steel rod with two fixed supporting prisms P_1 and P_2 made of steel and a steel lentil A between them (see Fig. 142.1). Another lentil B is placed on one of the rod's ends (not between the prisms); it can be moved along the rod and fixed in a required place. By moving this lentil, one can make the two periods of oscillations equal, when the points of suspension are the edges of the prisms P_1 and P_2 . When the equality of periods is achieved, the prisms' edges will be the respective points, and they will be asymmetric about the centre of mass C . Hence, when the equality of periods is achieved, the distance between the prisms is equal to the reduced length l

of the physical pendulum. Having measured its period of oscillations T , we can calculate g using Eq. (142.3).

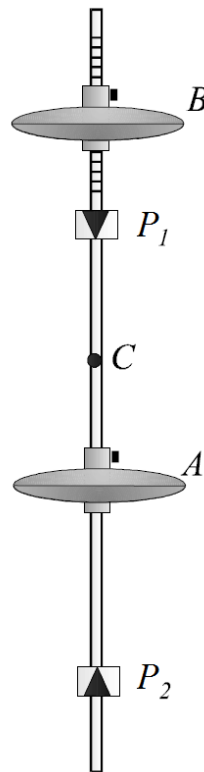


Figure 142.1 Kater's pendulum

Algorithm of measurements

1. Measure the distance l between the prisms with a ruler.
2. Suspend the pendulum by one of the prisms and deviate it by a small angle.
3. Count several total oscillations (the more, the better) and find the corresponding time t using the timer; calculate the period of oscillations T_1 .
4. Suspend the pendulum by another prism and find the period T_2 .
5. Repeat steps 3-4 (find T_1 and T_2 values) for 7-10 different positions of the lenticle B .
6. Plot the dependencies of the periods T_1 and T_2 versus the position of the lenticle B on a single plot.

7. Find the point of intersection of the plots and determine corresponding period for the respective points (where $T = T_1 = T_2$).
8. Calculate g using Eq. (142.3).
9. Analyse the results.

Questions

1. What are “inertia forces”?
2. Newton's law of universal gravitation.
3. Force of gravity, acceleration of free fall.
4. Draw a physical point in a reference frame bound to the rotating Earth. Which forces act on it? Estimate the contribution of the centrifugal force to the acceleration of free fall.
5. The concept of “weight.”
6. Describe the method for finding the value of g used in this work. Derive the formulas. Which approximations have been made in the construction of the experimental setup?

Part 2. MOLECULAR PHYSICS

211. Kinematic parameters of air molecules

Objective: determination of kinematic parameters of air molecules.

Tasks:

Acquaintance with Poiseuille's method for measuring the viscosity of a fluid.

Air viscosity measurement.

Estimation of the mean free path, frequency, and cross section of air molecules.

According to the kinetic theory of gases, a gas consists of many individual particles (molecules, atoms, ions). For simplicity, these particles can be considered as absolutely rigid spheres. Observations of Brownian motion allow suggesting that molecules are in constant motion. The high compressibility of gases indicates that their molecules are at a large average distance from each other. The question arises of estimating the kinematic parameters of this phenomenon. These are: the mean free path $\langle \lambda \rangle$ (i.e. the distance that a molecule travels between subsequent collisions with other particles) and the mean cross section (the effective collision area) $\langle \sigma \rangle = \pi \cdot \langle D \rangle^2$, where D is the distance between the centres of colliding molecules, at which their velocities change (the effective diameter of the molecule in the case of identical molecules or sum of molecular radii in case of different molecules). Mean cross section is a quantitative parameter that characterizes the intensity of collisions: the larger the cross section is, the more often collisions occur.

Obviously, the mean collisions' frequency $\langle \omega \rangle$ is proportional to the collision cross section. If the concentration of molecules is n (number of molecules in a unit volume) and the mean speed of the thermal motion of the molecules is $\langle v \rangle$, then

$$\langle \omega \rangle = \sqrt{2} \cdot \sigma \cdot n \cdot \langle v \rangle. \quad (211.1)$$

The additional factor takes into account the fact that the mean relative speed of molecules (with respect to each other) is $\sqrt{2}$ times greater than their mean speed $\langle v \rangle$ in the laboratory frame.

The mean free path $\langle \lambda \rangle$ can be derived as the average distance that a molecule moves per time unit (in fact, this value is the mean speed) divided by the number of collisions occurring during the same time:

$$\langle \lambda \rangle = \frac{\langle v \rangle}{\langle \omega \rangle} = \frac{1}{\sqrt{2} \cdot \sigma \cdot n} = \frac{1}{\pi \cdot \sqrt{2} \cdot (D)^2 \cdot n}. \quad (211.2)$$

Thus, simple mechanistic concepts make it possible to relate the microscopic parameters of a gas to each other.

On the other hand, the correlation between the dynamical viscosity η and microscopic parameter can be found provided that the Maxwell–Boltzmann distribution for the speed takes place:

$$f(v) = \left(\frac{2}{\pi}\right)^{1/2} \cdot \left(\frac{m}{kT}\right)^{3/2} \cdot v^2 \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot kT}\right),$$

where f is the fraction of the particles with a speed v , m is the particle mass, $k = 1.381 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$ is the Boltzmann constant, and T is the absolute temperature (K). In this case, we can write down:

$$\eta = \frac{1}{3} \cdot \rho \cdot \langle \lambda \rangle \cdot \langle v \rangle, \quad (211.3)$$

where ρ is the density of the gas.

The Maxwell–Boltzmann distribution is true at equilibrium state, which predicts that the average speed of molecules is

$$\langle v \rangle = \sqrt{\frac{8 \cdot R \cdot T}{\pi \cdot \mu}}, \quad (211.4)$$

where $R = 8.3145 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ is the ideal gas constant, μ is the molar mass of the substance.

Thus, we can express the mean free path as a function of macroscopic parameters:

$$\langle \lambda \rangle = \frac{3 \cdot \eta}{\rho \cdot \langle v \rangle} = \frac{3 \cdot \eta}{\rho} \sqrt{\frac{\pi \cdot \mu}{8 \cdot R \cdot T}}. \quad (211.5)$$

So, knowing the molecular mass of air and measuring its viscosity and absolute temperature, we can calculate the free path $\langle \lambda \rangle$ using Eq. (211.5), and then estimate the mean cross section from Eq. (211.2).

Experimental setup

The idea of the experiment is based on the Poiseuille's equation for the volumetric flow rate Q which passes as a laminar flow through a cylindrical pipe of radius r and length l under the action of a constant pressure drop Δp at the ends of the pipe:

$$Q = \frac{\pi \cdot r^4}{8 \cdot \eta \cdot l} \cdot \Delta p . \quad (211.6)$$

Thus, to find the viscosity of a gas η , it is necessary to measure its volumetric flow rate through a capillary of known length and inner radius together with the pressure drop, and then:

$$\eta = \frac{\pi \cdot r^4}{8 \cdot l} \cdot \frac{\Delta p}{Q} . \quad (211.7)$$

Experiment is carried out on a setup drawn in figure 211.1. Capillary pipe T is attached to the vessel V and a water manometer (pressure gauges) M. Water is released through a regulated valve R1 in order to create the needed pressure difference Δp on the ends of the capillary (the valve R2 should be closed). Created rarefaction is monitored by the manometer M ($\Delta p = \rho_w \cdot g \cdot \Delta h$, where ρ_w is the density of water at the ambient temperature, g is the free fall acceleration, and Δh is the difference of the water levels in the manometer). As a result, air will enter the system through the capillary and come into the vessel V. The volume of water flowing per time unit is determined by a gauge glass and a timer; this value is equal to the volume of air passing through the pipe T at the same time. Valve R2 with a funnel is used to return water from the gauge glass back to the vessel before a new measurement.

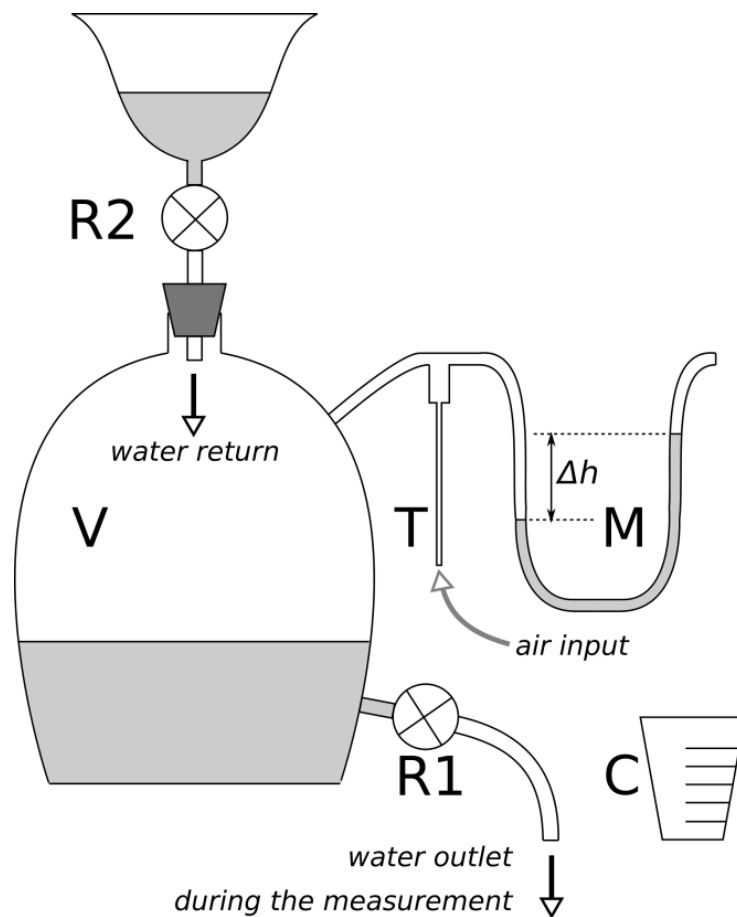


Fig. 211.1. Scheme of the setup. Auxiliary vessel is not shown

Algorithm of measurements

1. Put an auxiliary vessel under the valve R1 and smoothly open the valve to create the maximal possible pressure drop Δh on the manometer. Wait until the flow stabilizes and the reading from the manometer stop changing quickly; measure Δh_1 . Do not touch the valve now and replace the vessel with the measuring cup C. Measure the duration of the flowing of a certain portion of water V and the pressure difference Δh_2 which will be shown by the manometer at the end of this process. Close the valve R1. (N.B. The duration of the process must exceed 5 s due to the limited human reaction time.) Pour the water from the measuring cup and the auxiliary vessel back into the funnel inserted to the valve R2. Water can be returned from the funnel to the vessel V after the funnel is filled to half of its height.

2. Calculate the average pressure drop at the ends of the capillary during the process:

$$\Delta p = \rho_w \cdot g \cdot \frac{\Delta h_1 + \Delta h_2}{2} .$$

3. Measure the water temperature (or room temperature) to find its density.
4. Calculate the volumetric flow rate of air as $Q = V/t$.
5. Repeat steps 1-4 with several (8 to 10) different Δp values, big or small, to plot Q (vertical axis) vs. Δp (horizontal axis).
6. Analyse the obtained dependence of Q vs. Δp . Does it correspond to the Poiseuille's Eq. (211.6)? At the limit of small Δp 's, the dependence should be linear, and $Q(0) = 0$. Otherwise, the measurements should be repeated more accurately.
7. Calculate the average viscosity of air using your experimental data and Eq. (211.7). The capillary dimensions (l and r) are indicated on the setup.
8. Calculate the mean free path of air molecules using Eq. (211.5).
9. Measure atmospheric pressure with the barometer (if there is no barometer, take the value of mean sea-level pressure).
10. Calculate the mean cross section from Eq. (211.2). The concentration (number of molecules per unit volume) can be found as $n = p/kT$.
11. Estimate the diameter of an air molecule and the mean collisions' frequency.

Questions

1. Correlation between microscopic and macroscopic parameters of gases. The basic equation of the kinetic theory of gases.

2. How is the phenomenon of viscous flow in gases explained within the framework of kinetic theory of gases?

3. How does the viscosity of a gas depend on temperature and pressure?

4. How do the mean free path and the mean collisions' frequency depend on temperature and pressure?

5. Physical meaning of the mean cross section.

6. How big is the average speed of thermal motion of air molecules at room temperature? What is the mean collisions' frequency under these conditions?

7. Why does the indication Δh from the manometer decrease as the liquid flows out of the vessel V?

212. The adiabatic index and heat capacity of air at constant volume

Objective: measurement of the adiabatic index and molar heat capacity of air at constant volume.

Tasks:

Calculation of the adiabatic index according to the kinetic theory of gases.

Acquaintance with the theoretical foundations of method for measuring the adiabatic index.

Estimation of the number of degrees of freedom of air molecules.

The adiabatic index (also called Laplace's coefficient) γ is the ratio of the molar heat capacity at constant pressure, C_p , to the molar heat capacity at constant volume, C_v :

$$\gamma = \frac{C_p}{C_v} . \quad (212.1)$$

This index appears in the relation describing an adiabatic process:

$$pV^\gamma = \text{const} . \quad (212.2)$$

Since the Mayer's relation is fulfilled for ideal gases:

$$C_p - C_v = R , \quad (212.3)$$

then measurement of γ allows to find the molar heat capacity at constant volume as

$$C_v = \frac{R}{\gamma - 1} . \quad (212.4)$$

Having measured γ and using the conclusions of the kinetic theory of gases, we can obtain the information about the number of excited degrees of freedom i at a given temperature because

$$\gamma = \frac{C_p}{C_v} = \frac{i + 2}{i} , \quad (212.5)$$

and thus make assumption about the number of atoms that make up the air molecules.

Experimental setup

In what follows, we will use the assumption that air at room temperature exhibits the properties of an ideal gas. The setup is shown in figure 212.1. Glass vessel A (with a bag of silica gel inside to keep air dry) can be vented to the atmosphere through valve B. Manometer (pressure gauge) C measures the gauge pressure (pressure in the vessel above or below atmospheric pressure). Initially, when valve B is open, the air in the vessel is at atmospheric pressure p_0 and room temperature T_0 . If a small amount of air is quickly pumped into vessel A and immediately after that valve B is closed, the pressure and temperature in the vessel will increase.

Since there is a heat flow through the walls of the vessel, after a while the temperature inside the vessel will become equal to the ambient (room) temperature, and the pressure will slightly decrease to a new value

$$p_1 = p_0 + h_1, \quad (212.6)$$

where h_1 is the manometer reading. Let's call this gas state with $T_1 = T_0$ and pressure p_1 the first state.

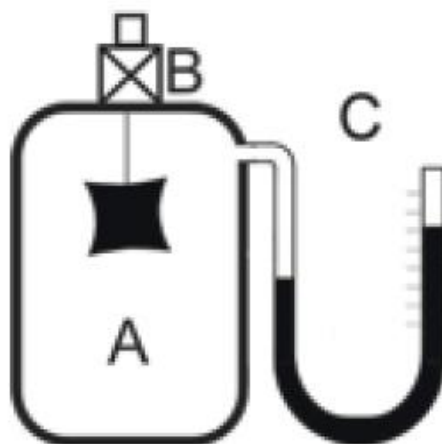


Fig. 212.1. Scheme of the setup

If the valve is opened quickly, the air in the vessel A will experience adiabatic expansion, the pressure will equal with the atmospheric pressure, and the temperature will fall to a new value T_2 . This will be the second state with parameters T_2 and $p_2 = p_0$.

If the valve is closed immediately after that (when the pressure reaches the value p_2), then a process will begin in which the temperature inside will grow and, finally, will reach T_0 , and the pressure will become equal to the new equilibrium value p_3 . This will be the third state with parameters $T_3 = T_0$ and

$$p_3 = p_0 + h_3, \quad (212.7)$$

where h_3 is the pressure manometer reading in the third state of the gas.

The laws of the ideal gas are formulated for a fixed amount of the gas. Therefore, we will consider an imaginary volume of the gas never leaving the vessel in the following discussion. For this volume we can write down the relations (212.8) and (212.9), taking into account that the transition from state 1 to state 2 is the adiabatic process, and the transition $2 \rightarrow 3$ is the isochoric process:

$$\frac{p_1^{\gamma-1}}{T_1^\gamma} = \frac{p_2^{\gamma-1}}{T_2^\gamma} \quad (212.8)$$

and

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}. \quad (212.9)$$

Combining Eqs. (212.6) and (212.8), we obtain:

$$\left(\frac{p_0 + h_1}{p_0}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)^\gamma \quad \text{or} \quad \left(1 + \frac{h_1}{p_0}\right)^{\gamma-1} = \left(1 + \frac{T_1 - T_2}{T_2}\right)^\gamma.$$

Since $h_1/p_0 \ll 1$ and $(T_1 - T_2)/T_2 \ll 1$, then both parts of the above equation can be represented as an expansion in an infinite series $(1 + x)^\alpha \approx 1 + \alpha \cdot x + \dots$, and the first two terms of the series will give a fairly good approximation:

$$1 + (\gamma - 1) \frac{h_1}{p_0} = 1 + \gamma \frac{T_1 - T_2}{T_2} \quad \text{and hence} \quad p_0 \frac{T_1 - T_2}{T_2} = \frac{\gamma - 1}{\gamma} h_1.$$

The left-hand side in the last equation is actually h_3 . For clarity of this statement, we substitute p_3 from Eq. (212.7) into (212.9), and then express h_3 in terms of the remaining parameters, we obtain:

$$h_3 = p_0 \frac{T_1 - T_2}{T_2} \quad \text{and finally} \quad h_3 = \frac{\gamma - 1}{\gamma} h_1.$$

The working formula is easily derived from the latter expression:

$$\gamma = \frac{h_1}{h_1 - h_3}. \quad (212.10)$$

Thus, to find the adiabatic index using this method, we do not need to calibrate the manometer; the only requirement is that the calibration dependence $h(p)$ be linear.

Algorithm of measurements

1. Check that the U-shaped manometer is filled with water. Open the valve B and wait for 2-3 minutes. Attach the rubber bulb to the outlet of the valve B. Pump air into the vessel by a quick squeezing of the bulb and close the valve. Repeat the pumping again so that the liquid level difference h in the manometer reaches 20-40 cm.
2. Wait for the pressure to stabilize due to heat conductivity (usually it takes about 5-6 minutes) and read the value h_1 .
3. Open the valve B and close it after the hissing of escaping air is heard no longer. Wait until the pressure to stabilize. Read h_3 value.
4. Repeat steps 1-3 at least 8 times.
5. Calculate γ using Eq. (212.10) and estimate the uncertainty (error) assuming that this value is the result of direct measurement.
6. Calculate the molar heat capacities of air C_V and C_p using Eqs. (212.4) and (212.3).
7. Find the number of degrees of freedom for air according to Eq. (212.5).
8. Draw conclusions about the correctness of kinetic theory of gases and quantum concepts about composition of air molecules.

9. Draw conclusions about the number of atoms in an air molecule.

Questions

1. Heat capacity. The value of capacity at constant volume.
2. The concept of the number of the freedom degrees of a molecule.
3. The adiabatic index and its relation to the number of the degrees of freedom.
4. Mayer's law.
5. Which of the relations written here are correct for ideal gas only? Which of them are fulfilled for a fixed amount of gas?
6. Present in a qualitative manner the processes in the gas occurring at each stage of the experiment on one diagram. Write down corresponding equations.
7. What condition must be met for the transition from the state 1 to state 2 to be an adiabatic process?
8. Why is it recommended waiting a few minutes before reading the pressure gauge? What if you do not follow this rule?
9. Is the ratio of the volumes of the rubber bulb and the vessel important?
10. What requirements must the vessel fulfil? Think about the volume, thickness, and rigidity of the wall, the colour and transparency of the walls, the shape of the vessel.
11. Analyse the relative uncertainty (error) of the measurements of γ and C_V . Draw conclusions.

221. The surface tension

Objective: determination of the surface tension of a liquid by the method of detaching droplet.

Tasks:

Acquaintance with the theory of the phenomenon of surface tension.

Measurement of surface tension by the detaching droplet method.

The phenomenon of surface tension is associated with the interaction between molecules. Figure 221.1 shows that the molecules in the volume and the molecules on the surface experience different effects from their neighbours, since the number of neighbouring molecules in these two cases is obviously different. Note that on the other side of the boundary (interface) there may be another material, such as air, but the interaction between molecules of different types is not the same as between molecules of the same type. The resulting force is directed tangentially to the liquid surface.

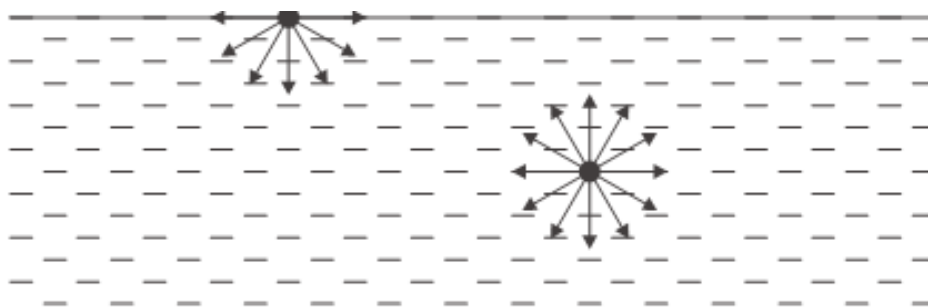


Fig. 221.1. Forces acting on molecules on the surface and in the volume of a liquid

The surface tension coefficient can be expressed in terms of force or in terms of energy. For example, the ratio of the change in free energy to

the corresponding change in surface area (at constant temperature) is equal to the surface tension of the liquid: $\sigma = \Delta E / \Delta S$.

Surface tension influences the process of drop falling from an outlet of a vessel in the gravitational field. The larger the coefficient σ , the more massive the drop will be detached from the outlet.

Figure 221.2 shows the process of drop formation. Before the moment of separation, a "neck" appears. This word denotes a region of a liquid that has parallel boundaries and a smaller diameter than the diameter of the drop itself. The force of surface tension acts along the neck; at the moment of detachment, it is equal to the weight of the drop. If the neck diameter is D , then the resulting surface tension force is $\pi \cdot D \cdot \sigma$. Therefore, the drop separation condition can be written as:

$$P = \pi \cdot D \cdot \sigma. \quad (221.1)$$

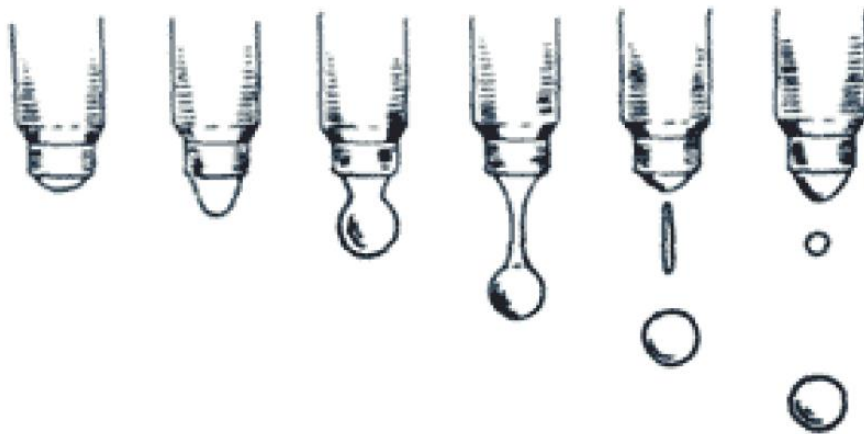


Fig. 221.2 Drop formation process

The surface tension coefficient of a liquid can be calculated by measuring the weight P of the fallen drop and the diameter of the neck at the moment of detachment.

It is known from observations that if drops of various liquids fall from a thin spout (capillary), then the diameter of the neck becomes the same for all liquids, provided that the diameter of the spout is small enough. In this case, Eq. (221.1) for two different liquids 1 and 2 gives the relation

$$\frac{\rho_1 V_1}{\sigma_1} = \frac{\rho_2 V_2}{\sigma_2} ,$$

where V_i are the volumes of various liquid drops, and ρ_i are their densities. Thus, if the surface tension σ_s is known for some liquid (let's call it "standard liquid"), then for any other liquid we can write:

$$\sigma = \frac{\rho V}{\rho_s V_s} \sigma_s = \frac{\rho n_s}{\rho_s n} \sigma_s ,$$

where n_s and n are the number of drops that can be formed from the same volumes of the standard and investigated liquids, respectively. Therefore, if there is a spout from which drops with the same neck diameters fall, it can be calibrated to measure surface tension. The value $K = n_s \cdot \sigma_s / \rho_s$ should first be determined from an experiment with a fixed volume V of a liquid with known σ_s and ρ_s (e.g., water), in which the number of drops n_s should be counted. Then the same amount V of the investigated liquid with a density ρ should be poured through the spout and count the corresponding number of drops n . Then the surface tension of the investigated liquid can be calculated by the expression:

$$\sigma = \frac{\rho n_s}{\rho_s n} \sigma_s = \frac{\rho}{n} K . \quad (221.2)$$

Algorithm of measurements

1. Wash (rinse) the burette with a thin spout with distilled water.
2. Fill the burette with distilled water (standard liquid) up to half its height.
3. Open the valve slightly so that water drips slowly.
4. Count the number of drops n corresponding to 1 ml of spilled water.
5. Repeat steps 3 and 4 three or four times and find the arithmetic mean $\langle n \rangle$.
6. Pour water from the burette and wash (rinse) it with a small amount of ethanol (or other investigated liquid).

7. Fill the burette with ethanol and repeat the measurements of n in a same way (steps 3–5).
8. Calculate the value of σ for ethanol.

Questions

1. Physical meaning of the surface tension coefficient. Why is it necessary to introduce this concept and to measure it?
2. Do the properties of an ambient gas or other liquid (assuming the liquids do not mix with each other) have an influence on the surface tension coefficient?
3. How does the surface tension depend on the temperature? Why?
4. Laplace pressure. Derivation of formulas.
5. How is surface tension related to the height of liquid rising in a capillary?
6. Why does a small drop appear between the main large drop and the spout at the moment of detachment?
7. How does viscosity affect the process of drop detachment?
8. Is it possible to determine the temperature dependence of the surface tension of a liquid using the method described here?

222. Measuring viscosity of liquid by the Stokes' method

Objective: measurement of viscosity of a liquid.

Task: Acquaintance with the theoretical basis of the Stokes' method.

A spherical body (hard ball or drop of liquid) of radius r falling in a liquid medium (viscosity η) with a speed of v experiences the force of gravity F_g , the upward buoyant (Archimedes') force F_A , and the resisting force of the medium F_r :

$$F_g = mg = V\rho_b g, F_A = \rho Vg, \text{ and } F_r = 6\pi \cdot \eta \cdot r \cdot v. \quad (222.1)$$

Here V is the ball's volume, ρ_b is its density, ρ is the density of the medium, and g is the free fall acceleration. The equation for F_r is called the Stokes' law. It was derived by G. Stokes assuming that (1) the Reynolds number, Re , is small (the flow is laminar), (2) liquid fills the whole space, and (3) liquid wets the ball (i.e., the closest molecular layer of the liquid moves with the ball).

O. Reynolds famously studied the conditions in which the flow of fluid transitioned from laminar flow to turbulent flow:

$$Re = \frac{\rho \cdot v \cdot L}{\eta},$$

where L is a characteristic linear dimension. For flow in a circular pipe, L is exactly equal to the inside pipe diameter. For a sphere in a fluid, L is the diameter of the sphere. Laminar flow occurs at low Reynolds numbers ($Re \ll Re_{\text{critical}}$), where viscous forces are dominant, and is characterized by smooth, constant fluid motion. Turbulent flow occurs at high Reynolds numbers ($Re \gg Re_{\text{critical}}$) and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities. The critical Reynolds number, Re_{critical} , is different for every geometry. For flow in a circular pipe $Re_{\text{critical}} = 2300$, for a sphere in a fluid $Re_{\text{critical}} = 10$.

The speed of the ball falling in a liquid changes as

$$v(t) = \frac{g \cdot V \cdot (\rho_b - \rho)}{6\pi \cdot \eta \cdot r} \left(1 - \exp\left(\frac{6\pi \cdot \eta \cdot r}{m} \cdot t\right) \right) . \quad (222.2)$$

Evidently, it tends exponentially to a certain limit value:

$$v_\infty = \frac{g \cdot V \cdot (\rho_b - \rho)}{6\pi \cdot \eta \cdot r} . \quad (222.3)$$

The time characterising the increase in the speed of the ball is called the relaxation time:

$$\tau = \frac{m}{6\pi \cdot \eta \cdot r} . \quad (222.4)$$

Behind several τ periods, the ball's speed can be considered constant and equal to the limit value.

After substituting the expression for the volume of the ball $V = \frac{4 \cdot \pi \cdot r_b^3}{3}$ into Eq. (222.3), we get:

$$\eta = \frac{2g \cdot r_b^2 \cdot (\rho_b - \rho)}{9 \cdot v_\infty} . \quad (222.5)$$

Thus, the coefficient of the internal friction (viscosity) can be found experimentally, if we know the values of r_b , ρ_b and ρ and measure v_∞ . In practical measurements, the ratio of the ball's radius r_b to the radius R of the pipe with the liquid should be taken into account as follows:

$$\eta = \frac{2g \cdot r_b^2 \cdot (\rho_b - \rho)}{9v_\infty \cdot \left(1 + 2.4 \cdot \frac{r_b}{R}\right)} . \quad (222.6)$$

If we use a drop of liquid in another liquid, the viscosity of the liquid of which the drop is made, η_d , should also be taken into account. To do this, we introduce an additional factor into Eq. (222.6):

$$\beta = \frac{1 + \eta_d / \eta}{2/3 + \eta_d / \eta} ,$$

so that finally

$$\eta = \beta \frac{2g \cdot r_b^2 \cdot (\rho_b - \rho)}{9v_\infty \cdot \left(1 + 2.4 \cdot \frac{r_b}{R}\right)} .$$

If the ratio η_d/η is small (for example, if we consider a drop of water propagating through oil), we can assume that $\beta = 3/2$.

Experimental setup:

Cylindrical glass vessel with level marks, filled with investigated transparent oil; burette; distilled water; a lid for a vessel in the form of a cap; timer; ruler.

Algorithm of measurements

1. Measure the inner radius R of the cylinder and the distance between the level marks.
2. Put on the cap.
3. Fill the burette with distilled water.
4. Carefully open the valve and find the position at which the drops fall into the cap slowly, one by one.
5. Count how many drops are in one millilitre of water. From this value, find the volume and radius (r_b) of an individual drop.
6. Switch on the illumination behind the vessel.
7. Remove the cap and let one drop fall into the oil. Return the cap to its place. Measure the time it takes for the ball to pass between the two marks. Check if the speed is constant by comparing the times of passage of two intervals drawn on the pipe.
8. Calculate the speed of fall (between the two marks at the biggest distance). Repeat this measurement 6–10 times and find the average value of the speed v_∞ .
9. Calculate the viscosity of oil using Eq. (222.7). The density of the oil is written on the setup. Estimate the uncertainty (error).
10. Calculate the relaxation time using Eq. (222.4).
11. Calculate the resistance force of the medium using the Stokes' law.
12. Calculate the Reynolds number.

Questions

1. Internal friction in liquids.
2. Dynamic and kinematic viscosity (physical meaning).
3. Stokes' method of measuring the viscosity.
4. What is the shape of a drop of water as it passes through oil?

223. Measuring viscosity of liquid by the Stokes' method, variant 2

Objective: measurement of viscosity of a liquid.

Task: Acquaintance with the theoretical basis of the Stokes' method.

A spherical body (hard ball or drop of liquid) of radius r falling in a liquid medium (viscosity η) with a speed of v , experiences the force of gravity F_g , the upward buoyant (Archimedes') force F_A , and the resisting force of the medium F_r :

$$F_g = mg = V\rho_b g, \quad F_A = \rho Vg, \quad \text{and} \quad F_r = 6\pi \cdot \eta \cdot r \cdot v. \quad (223.1)$$

Here V is the ball's volume, ρ_b is its density, ρ is the density of the medium, and g is the free fall acceleration. The equation for F_r is called the Stokes' law. It was derived by G. Stokes assuming that (1) the Reynolds number, Re , is small (the flow is laminar), (2) liquid fills the whole space, and (3) liquid wets the ball (i.e., the closest molecular layer of the liquid moves with the ball).

O. Reynolds famously studied the conditions in which the flow of fluid transitioned from laminar flow to turbulent flow:

$$Re = \frac{\rho \cdot v \cdot L}{\eta},$$

where L is a characteristic linear dimension. For flow in a circular pipe, L is exactly equal to the inside pipe diameter. For a sphere in a fluid, L is the diameter of the sphere. Laminar flow occurs at low Reynolds numbers ($Re \ll Re_{\text{critical}}$), where viscous forces are dominant, and is characterized by smooth, constant fluid motion. Turbulent flow occurs at high Reynolds numbers ($Re \gg Re_{\text{critical}}$) and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities. The critical Reynolds number, Re_{critical} , is different for every geometry. For flow in a circular pipe $Re_{\text{critical}} = 2300$, for a sphere in a fluid $Re_{\text{critical}} = 10$.

The speed of the ball falling in a liquid changes as

$$v(t) = \frac{g \cdot V \cdot (\rho_b - \rho)}{6\pi \cdot \eta \cdot r} \left(1 - \exp\left(\frac{6\pi \cdot \eta \cdot r}{m} \cdot t\right) \right). \quad (223.2)$$

Evidently, it tends exponentially to a certain limit value:

$$v_\infty = \frac{g \cdot V \cdot (\rho_b - \rho)}{6\pi \cdot \eta \cdot r}. \quad (223.3)$$

The time characterizing the increase in the speed of the ball is called the relaxation time:

$$\tau = \frac{m}{6\pi \cdot \eta \cdot r}. \quad (223.4)$$

Behind several τ periods, the ball's speed can be considered constant and equal to the limit value.

After substituting the expression for the volume of the ball $V = \frac{4 \cdot \pi \cdot r_b^3}{3}$ into Eq. (222.3), we get:

$$\eta = \frac{2g \cdot r_b^2 \cdot (\rho_b - \rho)}{9 \cdot v_\infty}. \quad (223.5)$$

Thus, the coefficient of the internal friction (viscosity) can be found experimentally, if we know the values of r_b , ρ_b and ρ and measure v_∞ . In practical measurements, the ratio of the ball's radius r_b to the radius R of the pipe with the liquid should be taken into account as follows:

$$\eta = \frac{2g \cdot r_b^2 \cdot (\rho_b - \rho)}{9v_\infty \cdot \left(1 + 2.4 \cdot \frac{r_b}{R}\right)}. \quad (223.6)$$

Experimental setup:

Cylindrical glass vessel with level marks, filled with the investigated transparent oil or glycerol; set of identical steel balls; timer; ruler.

Algorithm of measurements

1. Measure the inner radius R of the cylinder and the distance between the level marks.

2. Throw the ball into the vessel. Measure the time it takes for the ball to pass between the two marks. Check if the speed is constant by comparing the times of passage of two intervals drawn on the pipe.
3. Calculate the speed of fall (between the two marks at the biggest distance). Repeat this measurement 6–10 times and find the average value of the speed v_{∞} .
4. Calculate the viscosity using Eq. (223.6). The density of the liquid and the size of the ball are written on the setup. Estimate the uncertainty (error).
5. Calculate the relaxation time using Eq. (223.4).
6. Calculate the resistance force of the medium using the Stokes' law.
7. Calculate the Reynolds number.

Questions

1. Internal friction in liquids.
2. Dynamic and kinematic viscosity (physical meaning).
3. Stokes' method of measuring the viscosity.

224. Measuring viscosity by the Poiseuille's method

Objective: acquaintance with Poiseuille's method of measuring viscosity.

Tasks:

Learning theoretical basis, i.e., Poiseuille's equation.

Acquaintance with the Ostwald viscometer.

Measuring viscosity of ethanol by the Poiseuille's method.

The Poiseuille's method is based on determination of fluid flow through a capillary of known dimensions due to a certain pressure drop at the ends of the capillary.

Consider a cylindrical pipe with radius R and a length l . Let a liquid with a density ρ and a viscosity η flow through this capillary. The pressure drop at both ends of the pipe is Δp .

Imagine a cylindrical part of volume in a flowing liquid, which is coaxial with the pipe and has a radius r and the length l (the same as the pipe). It experiences an external force F_e due to the pressure drop:

$$F_e = \pi r^2 \cdot \Delta p .$$

In the case of a stationary flow (with no acceleration), this force is equal to the internal friction force, which is determined by the Newtonian law of viscosity:

$$F_{\text{if}} = -\eta \cdot S_{\text{side}} \cdot \frac{dv}{dr} = -\eta \cdot 2\pi \cdot r \cdot l \cdot \frac{dv}{dr}, \quad F_{\text{if}} = F_e = S_{\text{bas}} \cdot \Delta p = \pi r^2 \cdot \Delta p ,$$

where S_{side} and S_{bas} are the areas of the side wall and the basis of the imaginary volume element. From this relation, we derive the differential equation:

$$dv = -\frac{\Delta p}{2\eta \cdot l} \cdot r dr .$$

If the liquid wets the walls of the pipe, then we can assume that the layer closest to the wall is immobile. Thus, the flow speed changes

from 0 at the wall to some value v in the volume of the capillary, and the previous differential equation can be integrated as follows:

$$\int_0^v dv = -\frac{\Delta p}{2\eta \cdot l} \int_R^r r dr .$$

Thus, at a given distance r from the axis the liquid flows at the speed of

$$v = \frac{\Delta p}{4\eta \cdot l} \cdot (R^2 - r^2) = v_0 - \frac{\Delta p}{4\eta \cdot l} \cdot r^2 . \quad (224.1)$$

Obviously, v_0 is the speed in the central part of the capillary (on the axis).

If we now consider a thin tube-like layer with an inner radius r and an outer radius $(r + dr)$, then we can assume that liquid in it moves with a speed v (if the layer is thin enough, then we can neglect the fact that the speeds on its inner wall and the outer wall are slightly different). Then the volumetric flow rate (the mass of the liquid which flows through its cross-sectional area dS per time unit) can be calculated as

$$dQ = \rho \cdot v dS = \rho \frac{\Delta p}{4\eta l} \cdot (R^2 - r^2) \cdot (2\pi \cdot r dr) .$$

Then the volumetric flow rate through the entire pipe of radius R is equal to

$$Q = \frac{\pi \cdot \Delta p \cdot \rho}{2\eta \cdot l} \cdot \int_0^R (R^2 - r^2) \cdot r dr = \frac{\pi \cdot \Delta p \cdot \rho \cdot R^4}{8\eta \cdot l} = \frac{v_0 \cdot \rho \cdot S}{2} . \quad (224.2)$$

This expression is called Poiseuille's equation. Using this formula, the viscosity can be determined by passing the liquid through a pipe of length l and the radius R . The measured parameters are the pressure drop at the ends of the pipe Δp and the volumetric flow rate Q .

However, one nuance should be emphasized. We should be sure that internal friction has a significant effect on the flow value. Too small Q would make the measurement impractically slow; too fast Q will result in a turbulent flow rather than laminar flow. In the latter case all formulas above are not true. The minimal radius R is determined by the possibility

of manufacturing the thin glass pipe with a precisely known radius; it is on the order of 1 mm. The length should be about 100-1000 times larger than the diameter.

An Ostwald viscometer consists of two communicating glass vessels of variable diameter, into which the liquid is poured through the wide neck of the elbow B (see figure 224.1). The principle of working is based on measuring the time during which a standard and investigated liquid samples of the same volume pass through the same capillaries C. Using a rubber bulb, the liquid is pumped from the elbow B to the bulb A above the M1 mark, and then it is allowed to flow back due to the force of gravity. The time t required for the liquid meniscus to pass between the marks M1 and M2 is measured by a timer. Direct use of Poiseuille's equation is difficult because too many parameters should be known. Therefore, it is easier to measure the ratio of the viscosities of the investigated and the standard liquids, e.g., water (having the viscosity η_w and passing between the marks of the viscometer during the time t_w). Since the pressure drops at the capillary's ends are proportional to the liquid densities (ρ and ρ_w), we can write down:

$$\frac{\eta}{\eta_w} = \frac{(\pi \cdot \Delta p \cdot r^4)/(8V \cdot l)}{(\pi \cdot \Delta p_w \cdot r^4)/(8V \cdot l)} = \frac{\Delta p \cdot t}{\Delta p_w \cdot t_w} = \frac{\rho \cdot t}{\rho_w \cdot t_w},$$

or

$$\eta = \eta_w \cdot \frac{\rho \cdot t}{\rho_w \cdot t_w}.$$

Experimental setup

Thermometer, timer, two Ostwald viscometers.

An Ostwald viscometer is a fragile device. For this reason, two identical viscometers already filled with water and ethanol are placed in a transparent case. A rubber bulb can be attached to a viscometer through a hose.

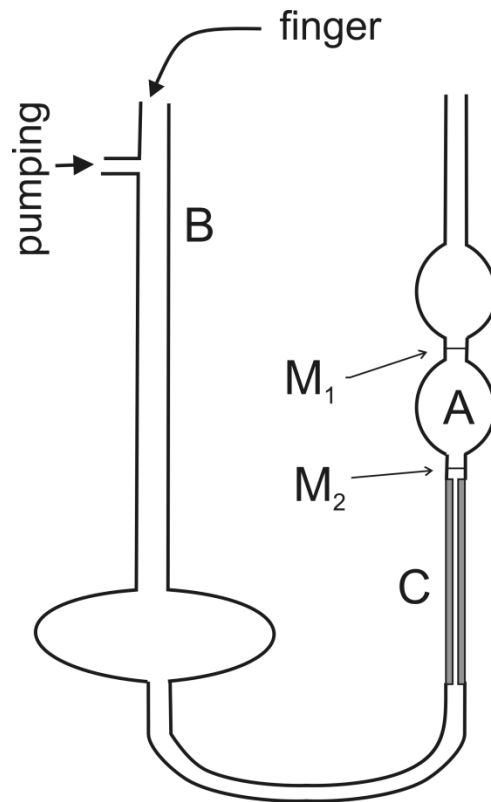


Fig. 224.1. Scheme of the setup

Algorithm of measurements

1. Close pipe B of the viscometer with your finger and pump the standard liquid (water) from the elbow B into the bulb A higher than the mark M_1 . To avoid rough inaccuracies, it is necessary that the bulb B is still partially filled with liquid. Also note that the hole above the bulb A is not blocked occasionally with a drop.
2. Remove your finger. Measure the time t_w during which water passes from mark M_1 to mark M_2 .
3. Repeat the measurements several times to find the mean value $\langle t_w \rangle$ and the uncertainty (error).
4. Repeat steps 1–3 for the investigated liquid (ethanol or isopropanol).
5. Measure the temperature in the room. Using reference tables, find the data for water and ethanol (ρ , ρ_w , η_w).
6. Calculate the viscosity of ethanol and estimate the uncertainty.

Questions

1. Internal friction in liquids. Newtonian law of viscosity.
2. Physical meaning of the coefficient of dynamic viscosity.
3. Newtonian and non-Newtonian liquids.
4. Which factors determine the friction force resisting the flow in a pipe?
5. Design of the Ostwald viscometer.
6. Estimate the pressure inside a syringe during injection.

Appendix

Tables:

Density of ethanol at different temperatures

$T, ^\circ\text{C}$	0	10	20	30	40	50	60	70	80	90	100
$\rho, \text{kg/m}^3$	812.1	801.4	790.5	779.3	768.0	756.4	744.6	732.4	720.0	707.3	694.2

Density of isopropanol (2-propanol) at different temperatures

$T, ^\circ\text{C}$	20	40	60	80
$\rho, \text{kg/m}^3$	786.5	769.2	750.2	729.3

Density and dynamic viscosity of water at different temperatures

$T, ^\circ\text{C}$	0	10	15	20	25	30	40	50
$\rho, \text{kg/m}^3$	999.82	999.77	999.19	998.29	997.13	995.71	992.25	988.02
$\eta, \text{mPa}\cdot\text{s}$	1.792	1.308	1.139	1.003	0.891	0.798	0.653	0.547

Viscosities of some liquids, $\text{mPa}\cdot\text{s}$ (at atmospheric pressure)

$T, ^\circ\text{C}$	0	20	50	70	100			
Glycerol	12100	1480	180	59	13			
Kerosene	2.2	1.5	0.95	0.75	0.54			
Oleum ricini		987	129	49				
Viscosity of alcohols								
$T, ^\circ\text{C}$	0	10	20	25	30	40	50	60
Ethanol	1.773	1.466	1.200	1.096	1.003	0.834	0.702	0.592
Isopropanol		3.308	2.381	2.012	1.724	1.326		

Viscosities of some gases, $\mu\text{Pa}\cdot\text{s}$ (at atmospheric pressure)

<i>T</i>, K	150	200	250	300	400
Nitrogen	10	12.9	15.5	17.9	22.1
Hydrogen	5.57	6.78	7.90	8.94	10.9
Air	10.3	13.2	16.0	18.5	23.0
CO₂		10.2	12.6	15.0	19.5

Viscosity of air ($\mu\text{Pa}\cdot\text{s}$) at different temperatures and pressure

<i>p</i>, bar	T, °C		
	0	25	100
1	17.2	18.37	21.0
20	17.53	18.65	22.02
50	18.15	19.22	22.40
100	19.70	20.60	23.35
200	23.70	23.95	25.30

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Efimov Sergey Vladimirovich
Turanov Aleksandr Nikolaevich

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