DIFFRACTION OF THE LOW-FREQUENCY EM FIELDS ON CONDUCTING EPS OBJECTS

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Abstract

A theoretical analysis of the contribution of the field of diffraction of electromagnetic waves by symmetric conducting objects to the total field of the VLF-VLF range near the sources, which can be individual elements of electric power systems, is presented. Quantitative estimates are given. The results obtained in the form of relations that are quite simple and convenient for practical use are of interest for solving a set of problems associated with various aspects of the problems of electromagnetic compatibility, noise immunity and life safety in the electric power industry.

Keywords: diffraction; electric power system (EPS); electric power industry; symmetric conducting objects; electromagnetic compatibility; noise immunity; analytical solution

1. Introduction

Study of the low-frequency (ELF-VLF frequency range – 10-10000 Hz) electromagnetic (EM) fields generated by the elements of the electric power systems (EPS), and investigation of the influence of the EM fields of natural and artificial origin onto different EPS structures are a great interest at solving the problems complex associated with different aspects of the electromagnetic compatibility, noise immunity and life safety in the electric power industry [1]. At this, the study of the structure and intensity of the EM field near objects of various, some-times quite complex, configurations (various construction objects, pipelines, open cable lines, etc.) requires an analysis of the overall picture, which is a superposition of the source field and the field resulting from diffraction at the appropriate object. The aim of this work is to theoretically analyze and obtaining quantitative estimates of the contribution of the diffraction field of EM waves on conducting objects of cylindrical and spherical shapes to the total field near the sources, which can be individual elements of the EPS. In our analysis, we will restrict ourselves to the assumptions: 1) the objects on which the EM field is studied are ideally conducting, that is permissible in conditions of low ground conductivity, that often occurs in real conditions; 2) the waves of the ELF-VLF frequency range are flat monochromatic, since a small region of the wave zone is considered.

2. Diffraction on cylindrical object

We will assume that in the case of low conductivity of the earth, in some approximation, it is permissible to reduce the problem to assessing the contribution of the field caused by diffraction on an ideally conducting cylinder of infinite length (the latter is true, since the resulting field is investigated at distances from the object much smaller than its linear dimensions).

We introduce a cylindrical coordinate system so that the *x*-axis coincides with the cylinder axis (see fig. 1). Suppose that for the incident wave $\mathbf{E}_0 \parallel \mathbf{x}$, $\mathbf{k} \perp \mathbf{x}$ (the task is to estimate the maximum contribution of the diffraction component), the angle α is measured from the direction **k**. In this case, the field has components E_x, H_r, H_α. Taking into account the geometry of the problem, define the function E_x from the wave equation for **E**: $\Delta\mathbf{E} - (1/v_{ph}^2)(\partial^2\mathbf{E}/\partial t^2) = 0$. The component E_{xc} of the secondary field due to diffraction, $\mathbf{E}_c = \mathbf{E} - \mathbf{E}_0$, satisfies the differential equation

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$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_c}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 E_c}{\partial\alpha^2}+k^2E_c=0. \tag{1}$$

By separating the variables in (1), we obtain [2]

$$E''_\nu(r)+(1/r)E'_\nu(r)+(k^2-\nu^2/r^2)E_\nu(r)=0, \tag{2}$$

$$E''_\nu(\alpha)+\nu^2E_\nu(\alpha)=0, \tag{3}$$

where $\nu^2 = C$ is a separation constant.

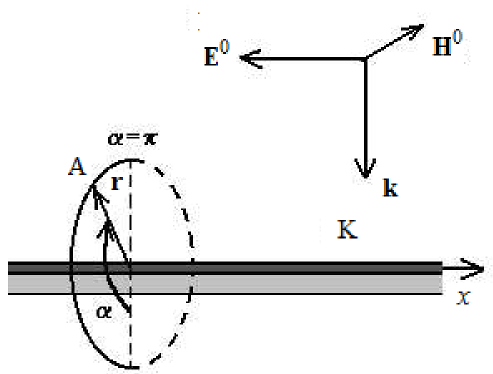

Fig. 1. Scheme of the statement of the problem of diffraction on a cylindrical object: A is a point, in which the field is calculated; K is a cylindrical object

Equation (2) is the Bessel equation, its solution can be written in the following form

$$E_c=C_1H_\nu^{(1)}(kr)+C_2H_\nu^{(2)}(kr),$$

moreover, the second term describes a converging wave, which does not exist in the real conditions described by the problem.

In the range of small values of r (near the object where $r << \lambda$), using boundary condition $r\to 0$, write function E_c in form of multiplication

$$E_c=\varepsilon_0 f_1(r)f_2(\alpha), \tag{4}$$

where $f_1(r)$ is a power-law function describing the attenuation of a wave with increasing r; $f_2(\alpha)$ is some function defining the oscillating distribution of vector $\mathbf{E}_c$ “within” the envelope – power function.

Since in the considering frequency range ($f\approx 10^1\div 10^4$ Hz) $k\approx 10^{-4}\div 10^{-7}$ and, accordingly, $(kr) << 1$, then in order to satisfy condition (4), as a solution of eq. (2), one should choose the Hankel function for small values of the argument [2, 3]:

$$E_\nu(r)=H_\nu^{(1)}(kr)=(1/i\pi)\,\Gamma(\nu)(kr/2)^{-\nu} \qquad (\nu\neq -1,-2,\ldots). \tag{5}$$

Equation (3) has solution $E_\nu(\alpha)=A_\nu\exp(i\nu\alpha)+B_\nu(\alpha)\exp(-i\nu\alpha)$. Parameter ν is integer and, according to (5), is positive. Taking into account that the general solution of eq. (1) is $E_c(r,\alpha)=\sum_\nu E_\nu(r)E_\nu(\alpha)$, we have

$$E_c(r,\alpha)=\varepsilon_0\sum_{\nu=0}^{\infty}\left(A_\nu\,\mathrm{e}^{\,i\nu\alpha}+B_\nu\,\mathrm{e}^{\,-i\nu\alpha}\right)H_\nu^{(1)}(kr).$$

The coefficients A_ν, B_ν can be easily found from the boundary condition on the surface of an ideally conducting cylinder $\mathbf{E}_c+\mathbf{E}_0=0$ at $r = a$ (where a is the radius of the cylinder), using the orthogonality property of $\exp(im\alpha)$:

$$A_\nu=i^\nu J_\nu(ka)/H_\nu^{(1)}(ka), \qquad B_\nu=i^\nu J_\nu(ka)/H_\nu^{(1)}(ka).$$

The total electric field is thus equal to

$$E_x(r,\alpha)=\varepsilon_0\left\{\exp\,(ikr\cos\alpha)-\sum_\nu \frac{J_\nu(ka)}{H_\nu^{(1)}(ka)}H_\nu^{(1)}(kr)\left(i^\nu\,\mathrm{e}^{i\nu\alpha}+i^{-\nu}\,\mathrm{e}^{-i\nu\alpha}\right)\right\}. \qquad (6)$$

Component H_α is found from the first Maxwell equation:

$$H_\alpha=-(1/ik)\,\partial E_x/\partial r=-\varepsilon_0\{\cos\alpha\exp\,(ikr\cos\alpha)-$$
$$-\sum_\nu \frac{J_\nu(ka)}{H_\nu^{(1)}(ka)}\frac{dH_\nu^{(1)}(kr)}{d(kr)}\left(i^{\nu-1}\,\mathrm{e}^{\,i\nu\alpha}+i^{-\nu-1}\,\mathrm{e}^{\,-i\nu\alpha}\right)\Bigg\}. \qquad (7)$$

Component H_r at $0\le\alpha\le\pi$ is calculated analogously.

The obtained relations (6) and (7) make it possible to estimate the contribution of the diffractive part to the total field received by the antenna:

$$\delta_{E_x}(r,k)=E_{xc}/(E_{x0}+E_{xc}),\qquad \delta_{H_\alpha}(r,k)=H_{\alpha c}/(H_{\alpha 0}+H_{\alpha c}).$$

The estimates of the effect of diffraction on a cylindrical object for waves with frequencies of 10÷10000 Hz at a = 1 cm, r = 1÷30 m made it possible to obtain the following analytical dependences of the relative estimates on the spatial coordinates and k:

$$\delta_{E_x}(r,k)\,|_{\alpha=\pi}=e^{\,ikr}(k/2r)^2,\qquad \delta_{H_\alpha}\,|_{\alpha=\pi}=ie^{\,ikr}k/2r^3. \qquad (8)$$

Formulas (8) make it easy to calculate the relative contribution of the diffraction component to the total EM field near a conducting cylindrical object.

3. Diffraction on spherical object

Let us now consider the contribution to the total EM field of the diffraction field on a spherical object. Let a homogeneous plane wave, the vectors of which have amplitudes

$$\dot E_m^{\,0}=x_0\,\dot A\,\mathrm{e}^{-i\dot k_0 z},\qquad \dot H_m^{\,0}=y_0\left(\dot A/\dot W_0\right)\mathrm{e}^{-i\dot k_0 z},$$

falls on a spherical body with the dielectric and magnetic constants $\dot\varepsilon$, $\dot\mu$; the constants of the medium are ε_0 and μ_0 (see fig. 2).

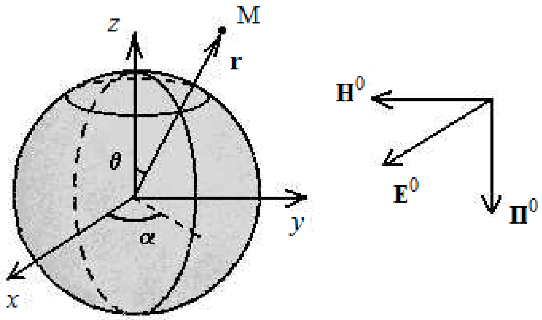

Fig. 2. Scheme for the statement of the diffraction problem on a spherical object, M is a point where the EM field is calculated.

It is necessary to construct solutions for the external diffraction field (we are not interested in the internal diffraction field in the considered formulation of the problem), which can be obtained by the same way as in the previous section, by expanding the incident wave into suitable functions and compiling similar expansions with undefined coefficients for the diffraction field. The latter will be found when the corresponding boundary conditions are imposed. Note, however, that the implementation of such an approach in the case of spherical geometry turns out to be more complicated.

We will use the spherical harmonics $\Re,\Theta,\Im$. The homogeneous Helmholtz equation describing the field in spherical coordinates has the form

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial u}{\partial\vartheta}\right)+\frac{1}{r^2\sin^2\vartheta}\frac{\partial^2 u}{\partial\alpha^2}+k^2u=0. \qquad (9)$$

Substitution $u(r,\vartheta,\alpha)=\Re(r)\Theta(\vartheta)\Im(\alpha)$ followed by multiplication of all terms of eq. (9) by

$r^2 \sin^2 \vartheta / (\Re\Theta\Im)$ gives

$$\frac{\sin^2\vartheta}{\Re}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Re}{\partial r}\right)+\frac{\sin\vartheta}{\Theta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Theta}{\partial\vartheta}\right)+\frac{1}{\Im}\frac{\partial^2\Im}{\partial\alpha^2}+k^2r^2\sin^2\vartheta=0.$$

Equating the third term, which depends only on α, to $-m^2$, we obtain two equations, one of which is divided by $\sin^2\vartheta$ [3]. Further, the terms depending only on *r*, are equated to p^2. As a result, we obtain three ordinary differential equations:

$$\frac{1}{\Re}\frac{d}{dr}\left(r^2\frac{d\Re}{dr}\right)+k^2r^2=0,\qquad \frac{\partial^2\Im}{\partial\alpha^2}+m^2\Im=0,$$

$$\frac{1}{\Theta\sin\vartheta}\frac{d}{d\vartheta}\left(\sin\vartheta\frac{d\Theta}{d\vartheta}\right)-\frac{m^2}{\sin^2\vartheta}=-p^2. \tag{10}$$

If $0\le\alpha\le 2\pi$, then $u(r,\vartheta,\alpha+2\pi)=u(r,\vartheta,\alpha)$, so that $m = 0, 1, 2, \dots$. Making the change $t=\cos\vartheta$ in the second equation of set (10) we obtain

$$\frac{d}{dt}\left[\left(1-t^2\right)\frac{d\Theta}{dt}\right]+\left(p^2-\frac{m^2}{1-t^2}\right)\Theta=0\qquad(-1<t<1).$$

The bounded solutions of the last equation are the adjoint Legendre functions $P_n^{(m)}(t)$ [in this case, the eigenvalues corresponding to these eigenfunctions are $p^2 = n(n+1)$] [4]:

$$T(t)=P_n^{(m)}(t)=(1-t^2)^{m/2}\frac{d^mP_n(t)}{dt^m}\qquad(n=0,1,2,\dots,\ m\le n),$$

where $P_n(t)$ are the Legendre polynomials [3]. Hence it follows that $\Theta(\vartheta)=P_n^{(m)}(\cos\vartheta)\quad(m=0,1,2,\dots)$.

First of the eqs. (10) after differentiating the expression in parentheses and performing substitutions $p=n(n+1)$, $\Re(r)=\rho(r)/\sqrt{kr}$, it is reduced to the form

$$\frac{d^2\rho}{dr^2}+\frac{1}{r}\frac{d\rho}{dr}+\left[k^2-\left(n+\tfrac{1}{2}\right)^2/r^2\right]\rho=0,$$

and this is nothing but the Bessel equation of order $n+1/2$ with respect to the functiona $\rho(kr)$ [3]. Thus, the solution of the first equation of set (10) is the function

$$\Re(r)=\frac{1}{\sqrt{kr}}\left[AJ_{n+1/2}(kr)+BN_{n+1/2}(kr)\right]=\frac{1}{\sqrt{kr}}\left[PH^{(1)}_{n+1/2}(kr)+QH^{(2)}_{n+1/2}(kr)\right].$$

Finally, the solutions of eq. (9) have the form

$$u(r,\vartheta,\alpha)=\frac{1}{\sqrt{kr}}\begin{Bmatrix}AJ_{n+1/2}(kr)+BN_{n+1/2}(kr)\\ PH^{(1)}_{n+1/2}(kr)+QH^{(2)}_{n+1/2}(kr)\end{Bmatrix}\times P_n^{(m)}(\cos\vartheta)\begin{Bmatrix}C\cos m\alpha+D\sin m\alpha\\ R\mathrm{e}^{im\alpha}+S\mathrm{e}^{-im\alpha}\end{Bmatrix}.$$

Using the well-known recurrence relations for the spherical and modified spherical Bessel functions of the first kind [3], we can now write down the solution for the external diffraction field [5]:

$$\dot{E}_m=\dot{A}\sum_{n=1}^{\infty}(-i)^n\frac{2n+1}{n(n+1)}\left(c_n^M M_{on}+ic_n^N N_{en}\right),\qquad r>R; \tag{11}$$

$$\dot{H}_m = -\frac{\dot{A}}{\dot{W}} \sum_{n=1}^{\infty} (-i)^n \frac{2n+1}{n(n+1)} \left(b_n^N M_{en} - i\, b_n^M N_{on} \right), \qquad r > R. \tag{12}$$

$$M_{\substack{o \\ e}n} = \sqrt{\frac{\pi}{2k_0 r}}\; J_{n+1/2}(kr) \left[\pm \vartheta_0 \frac{1}{\sin\vartheta} P_n^{(1)}(\cos\vartheta) \begin{matrix}\cos \\ \sin\end{matrix}\, \alpha - \alpha_0 \frac{d}{d\vartheta} P_n^{(1)}(\cos\vartheta) \begin{matrix}\sin \\ \cos\end{matrix}\, \alpha \right],$$

$$\begin{aligned} N_{\substack{o \\ e}n} = \frac{1}{k_0 r} \Bigg\{ & r_0 n(n+1) \sqrt{\frac{\pi}{2k_0 r}}\, H^{(2)}_{n+1/2}(k_0 r) P_n^{(1)}(\cos\vartheta) \begin{matrix}\cos \\ \sin\end{matrix}\, \alpha + \\ & + \vartheta_0 \sqrt{\frac{\pi}{2}} \left[\sqrt{k_0 r}\, H^{(2)}_{n+1/2}(k_0 r) \right]' \frac{d}{d\vartheta} P_n^{(1)}(\cos\vartheta) \begin{matrix}\sin \\ \cos\end{matrix}\, \alpha \pm \\ & \pm \alpha_0 \frac{1}{\sin\vartheta} \sqrt{\frac{\pi}{2}} \left[\sqrt{k_0 r}\, H^{(2)}_{n+1/2}(k_0 r) \right]' P_n^{(1)}(\cos\vartheta) \begin{matrix}\cos \\ \sin\end{matrix}\, \alpha \Bigg\}. \end{aligned}$$

The indices *o* and *e* in the last formulas correspond to the choice, respectively, of the upper and lower variants of the double sign and the trigonometric function. The coefficients in parentheses of formulas (11), (12) are defined by the expressions given in [5]. Formulas (11), (12) allow to calculate the field outside the sphere of radius *R* for the ELF-VLF waves.

4. Quantitative estimations

To obtain quantitative estimates of the contribution of the diffraction field to the overall field, we numerically simulated the diffraction using the program DIFFRACT [5] specially developed on the basis of the algorithms proposed in [3]. The examples of results of simulation of diffraction on the conductive cylinder for the incident EM waves with frequencies of 10–30 Hz and 1–3 kHz are shown in figs. 3 and 4, respectively. The diffraction field for a spherical conducting object is qualitatively the same: the moduli of its components outside the sphere decrease exponentially with increasing r, and, as in the case of diffraction on a cylindrical object, the amplitudes of the components near the object depend inversely with the frequency of the incident wave. However, in contrast to the case of diffraction on a cylinder, the contribution to the total field of the diffraction field on a spherical object under the same conditions is almost an order of magnitude larger.

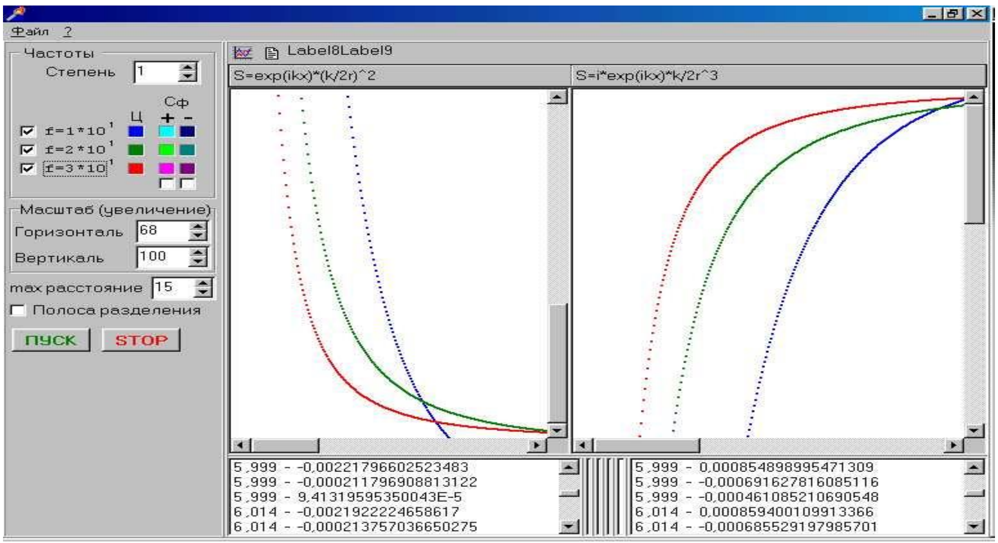

Fig. 3. Results of simulation of the contribution of the diffraction component to the total field in the frequency range f = 10÷30 Hz.

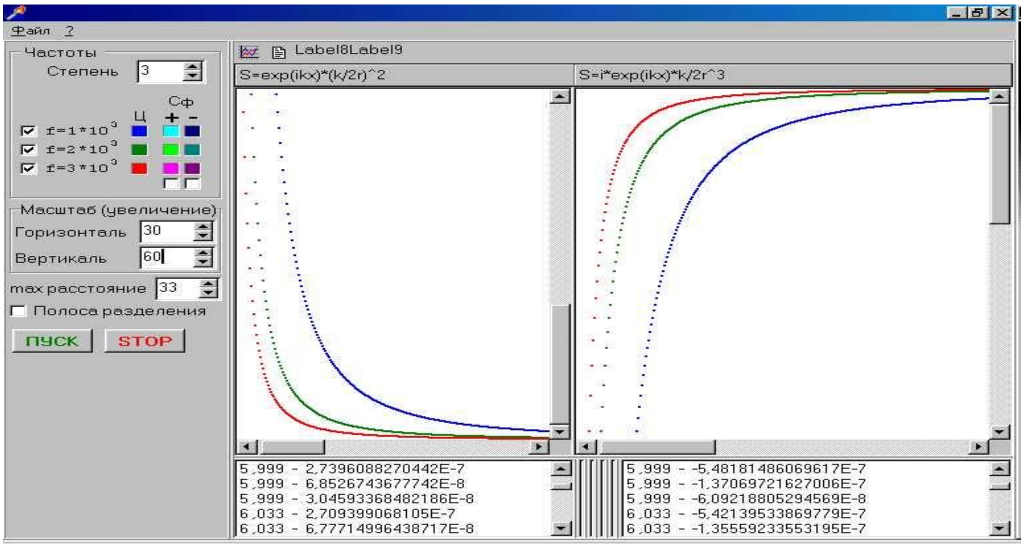

Fig. 4. The same as in fig. 3, for the frequency range $f = 1 \div 3$ kHz.

The results obtained, despite some idealization of the problem, allow us to conclude about some distortion of the low-frequency field as a result of diffraction, which increases with decreasing frequency according to laws (8), (11), (12), but these distortions, due to their relative smallness, should be taken into account only when constructing systems highly sensitive to external influences. An example of such a system is the mobile experimental complex developed by us for studying EM fields in a wide frequency range [6-8]. Using this complex, we measured EM fields near working electrical equipment at a number of energy and industrial enterprises, as well as near power lines of various voltages [9], at this the results of processing and analysis of experimental data generally confirmed our conclusions, the theoretical results and results presented in [10] for power lines with various design features.

5. Conclusion

In this paper, a theoretical analysis is carried out and quantitative estimates are given of the contribution of the field of diffraction of electromagnetic waves by symmetric conducting objects of cylindrical and spheroidal shapes to the total field of ELF-VLF range near such sources as some EPS elements. The results obtained for objects with a high degree of symmetry can become the basis for studying the structure and intensity of the EM field near objects of more complex configuration (buildings, pipelines, open cable lines, etc.), some parts of which are geometrically symmetric. In all cases, this requires an analysis of the total field, which is a superposition of the source field and the field that is the result of diffraction on the corresponding object or the superposition of the diffraction fields of its individual elements. The results are presented in the form of rather simple and convenient for calculations mathematical expressions and are of interest in solving problems related to various aspects of EMC problems, noise immunity and life safety in the electric power industry.

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