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# Классическая и современная геометрия

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The proof is based on Theorem 1, uses topological and stereometric methods, in particular, theorems on dividing a curve into congruent parts [3].

This study has something in common with the proof of the **hypothesis**: the only curves in  $A^n$ , any two oriented arcs of which are affinely equivalent, are enics of degree  $k = 1, \dots, n$ , i.e., curves defined in some affine coordinate system by the parametrization

$$(t, t^2, \dots, t^k, 0, \dots, 0), \quad t \in R.$$

(Here superscripts are degrees.) For  $k = 1, 2, 3$  this is, respectively, a straight line, a parabola, a cubic. So far, the hypothesis has been proven for  $C^n$ -smooth curves and in full for  $A^2$  [4].

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## INTRODUCTION TO MODIFIED GRAVITY

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The compact extra spaces is widely used idea [1, 2, 3, 4]. Any multi-dimensional model has to lead to the effective 4-dim theory. This would imply relations between the observable four-dimensional geometry and a geometry of the higher dimensions.

One of the question remaining not clarified yet is: why specific number of dimensions are compactified and stable while others expand? Which specific

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property of subspace leads to its quick growth? There are many attempts to clarify the problem, mostly related to introduction of fields other than gravity. It may be a scalar field (most used case) or gauge fields. A static solutions can be obtained using the Casimir effect or form fields. Sometimes one of the subspace is assumed to be Friedmann-Robertson-Walker space by definition. Another possibility was discussed in [5]: it was shown that if the scale factor of our 3D space is much larger than the growing scale factor of the extra dimensions, a contradiction with observations can be avoided.

The origin of our Universe is usually related to its quantum creation from the space-time foam. Here we are interested in the subsequent classical evolution of the metrics rather than a calculation of this probability. Manifolds are nucleated having specific metrics. The set of such metrics is assumed to be very rich. After nucleation, these manifolds evolve classically forming a set of asymptotic manifolds, one of which could be our Universe. In paper [6] we consider models of modified gravity acting in 5 and 6 dimensions. No other fields are attracted to stabilize an extra space. We have found out that a number of asymptotic solutions is quite limited. This conclusion was confirmed both analytically and numerically. There is a set of initial conditions that lead to a common asymptote of classical solutions.

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