

Rings with $x^n + x$ or $x^n - x$ nilpotent

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Received 28 July 2021

Accepted 31 August 2021

Published 25 October 2021

Communicated by P. Ara

Let R be a ring and let n be an arbitrary but fixed positive integer. We characterize those rings R whose elements a satisfy at least one of the relations that $a^n + a$ or $a^n - a$ is a nilpotent whenever $n \in \mathbb{N} \setminus \{1\}$. This extends results from the same branch obtained by Danchev [A characterization of weakly $J(n)$ -rings, *J. Math. Appl.* **41** (2018) 53–61], Koşan *et al.* [Rings with $x^n - x$ nilpotent, *J. Algebra Appl.* **19** (2020)] and Abyzov and Tapkin [On rings with $x^n - x$ nilpotent, *J. Algebra Appl.* **21** (2022)], respectively.

Keywords: Rings; fields; nilpotents; polynomials; Jacobson radical.

Mathematics Subject Classification 2020: 16U99, 16E50, 16W10, 13B99

1. Introduction and Background

Everywhere in the text of this paper, all our rings R are assumed to be associative, containing the identity element 1 which in general differs from the zero element 0 of R . Our standard terminology and notations are mainly in agreement with [16]. For

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