OntoMath^{Edu} Educational Mathematical Ontology: Prerequisites, Educational Levels and Educational Projections

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Abstract. The work is dedicated to development of prerequisite relationships of the educational mathematical ontology OntoMath^{Edu}. The concept *A* is called a prerequisite for the concept *B*, if a learner must study the concept *A* before approaching the concept *B*. OntoMath^{Edu} provides two approaches for defining prerequisite relationships: directly by establishing a relationship between concepts and indirectly by arrangement the concepts by educational levels. Prerequisite relationships and educational projections will be used in developing of digital mathematical educational platform of Kazan Federal University.

Keywords: Prerequisite, Ontology, Mathematical education, OntoMath^{Edu}.

1 Introduction

This work is dedicated to development of prerequisite relationships of the educational mathematical ontology OntoMath^{Edu} [1-3].

This ontology is intended to be a Linked Open Data hub for mathematical education, a linguistic resource for intelligent mathematical language processing and an end-user reference educational database. The ontology is organized in three layers: a foundational ontology layer, a domain ontology layer and a linguistic layer. The domain ontology layer contains language-independent math concepts from the secondary school mathematics curriculum. The concepts are organized in two hierarchies: a hierarchy of objects (such as *Line segment*, *Triangle*, *Inscribed polygon*, or *Pythagorean Theorem*) and a hierarchy of reified relationships (such as *Relationship between a tangent line and a circle*). The linguistic layer contains multilingual lexicons, providing linguistic grounding for the concepts from the domain ontology layer. The foundation ontology layer provides the concepts with meta-ontological annotations. The current version of

OntoMath^{Edu} contains 896 concepts from the secondary school Euclidean plane geometry curriculum.

OntoMath^{Edu} is a component of OntoMath digital ecosystem [4], an ecosystem of ontologies, text analytics tools, and applications for mathematical knowledge management, including semantic search for mathematical formulas [5] and a recommender system for mathematical papers [6]. OntoMath, in turn, underlines the Lobachevskii-DML digital mathematical library (https://lobachevskii-dml.ru/) [7] and a digital educational mathematical platform of Kazan Federal University under development.

For the ontology can be used for educational purposes, the logical relations between concepts must be complemented with the prerequisite ones. The concept *A* is called a prerequisite for the concept *B*, if a learner must study the concept *A* before approaching the concept *B*. For example, comprehension of the *Addition* concept is required to grasp the concept of *Multiplication*, and, more interesting, to grasp the very concept of *Function*, even though, from the logical point of view the later concept is more fundamental and is used in the definitions of the first two.

Prerequisite relationships are used in such tasks as automatic reading list generation [8], curriculum planning [9, 10], evaluation of educational resources [11] and prediction of academic performance [12].

2 Prerequisites, Educational Levels and Educational Projections

In contrast to logical relationships between concepts, prerequisite relationships are not universal and are relativized to particular education systems: given two concepts *A* and *B*, the prerequisite relation can hold between them in one education system, but doesn't hold in another. In particular, for the concept A that is prerequisite of the concept B in one education system, the following options are possible with respect to another education system:

- *A* is a prerequisite of *B* too. For example, *Circle* is a prerequisite of *Circumference* in both the Russian and the UK education systems.
- *A* is a prerequisite of *C*, and *C* is a prerequisite of *B*. For example, in UK education system, *Angle* is a prerequisite of *Alternate interior angles*, while in Russian education system, *Angle* is a prerequisite of *Alternate angles* and *Alternate angles* is prerequisite of *Alternate interior angles*.
- *B* is a prerequisite of *A*. There aren't examples of this pattern in the current version of OntoMath^{Edu}, but such pair of concepts can be *Set* and *Function*, or *Circle* and *Disk*.
- *A* isn't a prerequisite of *B*, because *A* and *B* are learned independently. For example, in the UK education system, *Plane motion* is a prerequisite of *Area of a polygon*, while in the Russian education system it isn't, because *Plane motion* and *Area of a polygon* are independent.

– *A* isn't a prerequisite of *B*, because *A* or *B* are not studied at all. For example, the prerequisite relation holds between *Angle* and *Complementary angles* concepts in the UK education system, but doesn't hold in the Russian education system, because the *Complementary angles* concept is not studied in it.

OntoMath[Edu] provides two approaches for defining prerequisite relationships: a direct and an indirect ones.

Direct approach. According to the direct approach, a prerequisite relationship is established directly between two concepts.

In order to relativize the relation to an education system, we intend using of "Descriptions and Situations" (D&S) design pattern, based on the top-level ontology DOLCE + DnS Ultralite [13-15]. However, manual annotation of D&S's is labour-intensive task.

As an alternative, the concepts can be linked by a subpropertis of the *has prerequisite* object property, corresponding to education systems, namely*: has prerequisite according to the Russian education system*, *has prerequisite according to the UK education system* and other.

Indirect approach. According to indirect approach, prerequisite relationships are established by arrangement of the concepts by educational levels.

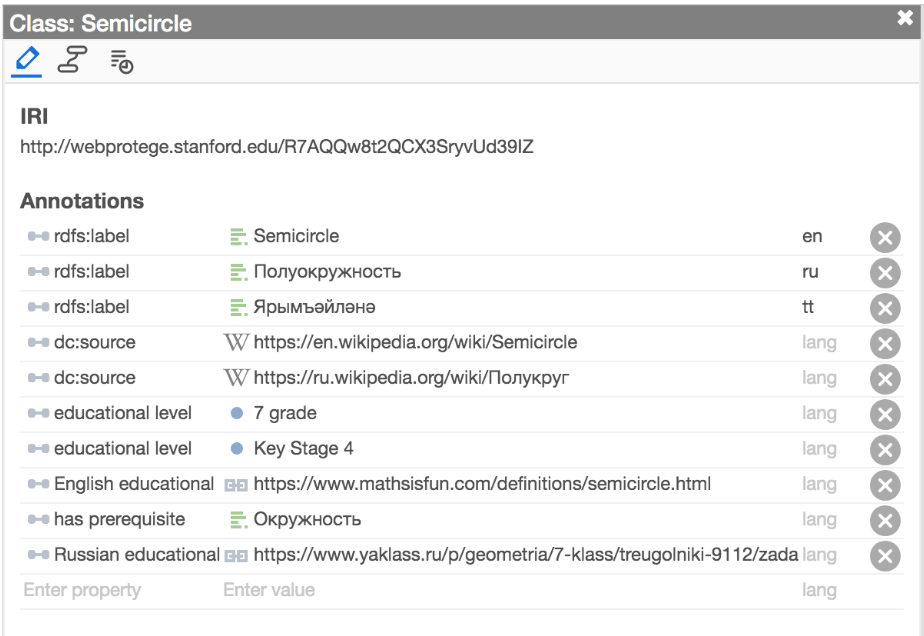

Fig. 1. The *Semicircle concept*, belonging to *7 grade* and *Key stage 4* education levels

Educational levels are the successive segments of the curriculum of an education system and roughly correspond to education grades. In the UK education system, the education levels are: *Key stage* 1 (1st–2nd years of study), *Key stage* 2 (3rd–6th years of study), *Key stage* 3 (7th–9th years of study), *Key stage* 4 (10th–11th years of study). In the Russian education system, the education levels are: *7 grade*, *8 grade*, *8 grade (extended)*, *9 grade*, *9 grade (extended)*, and *Additional program*.

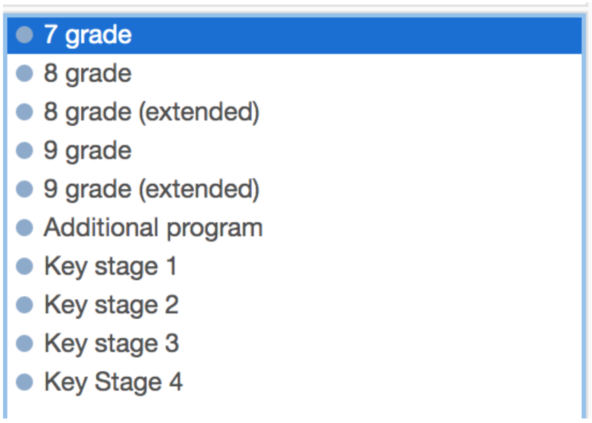

Fig. 2. Educational levels defined in OntoMath[Edu]

Every concept can belong to one education level of a given education system.

Just like concepts, educational levels are also related by prerequisite relation. The level *L1* is called a prerequisite for the level *L2*, if a learner must study the content of the level *L1* before approaching the content of the level *L2*. In terms of the direct prerequisite relationships between concepts, a prerequisite relationship between two levels can be interpreted as follows: if the level *L1* is called a prerequisite for the level *L2*, then for every concept, belonging to *L2* there is a prerequisite concept, belonging to *L1*.

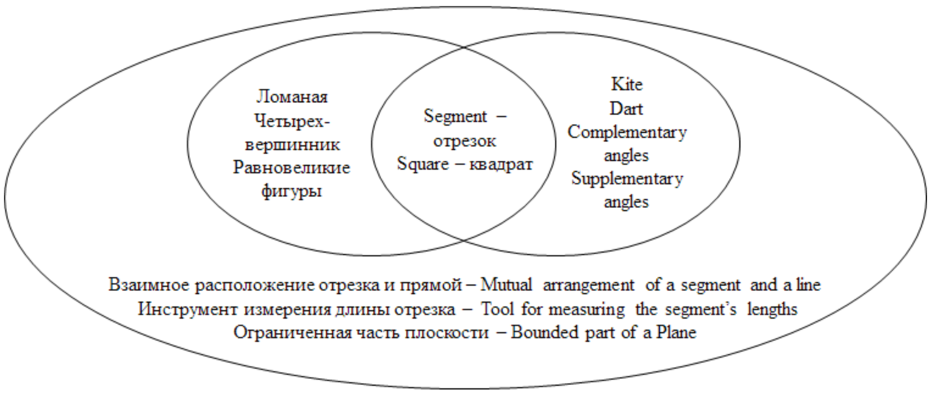

Fig. 3. Example of belonging of the concepts to the Russian (left circle) and the UK (right) education systems

Educational projections. Arrangement of concepts to educational levels allows to extract a projection of ontology to an education system (educational projection). An educational projection of the OntoMath^{Edu} ontology to education system *S* is a fragment of the ontology, containing all the concepts, that belong to educational levels of this education system. For example, the Russian education projection of OntoMath^{Edu} consists in the concepts, belonging to *grade 7*, *grade 8*, *grade 8 (extended)*, etc.

3 Conclusion

In this paper, we describe two approaches for defining prerequisite relationships: directly by establishing a relationship between concepts and indirectly by arrangement the concepts by educational levels. Arrangement of concepts by educational levels, in turn, allows to extract a projection of ontology to an education system (educational projection). Prerequisite relationships and educational projections will be used in developing of digital mathematical educational platform of Kazan Federal University.

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