

1 **Study of Seismo Effects in Electromagnetic**
2 **Field and Ionosphere**

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6 **Abstract.** The results of study of displaying of seismic activity in variations of
7 both the electromagnetic (EM) field and the basic ionospheric characteristics are
8 presented. The 3D case is considered taking into account the effects of weak non-
9 linearity, dispersion and dissipation in medium, that allows us to obtain more
10 accurate results for both the near and far zones from the earthquake epicenter. In
11 study of the seismic response in the EM field, it is shown that a precursor arises
12 ahead the front of seismic wave, its amplitude decreases exponentially with dis-
13 tance. Regarding the response at ionospheric heights, the seismo-ionospheric
14 post-effects have been studied, which are of great interest, in particular, for a
15 better understanding of relationships in the system "solid Earth – atmosphere –
16 ionosphere".

17 **Keywords:** Ionosphere, Electromagnetic Field, Earthquake, Precursor, Ray-
18 leigh Wave, IGW and TID Solitons.

19 **1 Introduction**

20 A number of experimental works starting from the 70s of the last century (see, e.g., [1,
21 2] and rather complete review [3]) reported the observation of low-frequency (LF) per-
22 turbations of the Earth's electromagnetic (EM) field a few seconds or minutes before
23 the appearance of a seismic wave at a registration point. As possible reasons for this
24 phenomenon, an assumption was made about the excitation in the *E*-layer of the iono-
25 sphere during an earthquake of a LF whistler propagating horizontally with a velocity
26 of ~ 20 km/s or slightly slower (10 km/s and lower [3]). Another possible physical
27 mechanism of the phenomenon is associated with the manifestation of the induction
28 seismomagnetic effect in the earth's crust. In theoretical papers [4, 5] it was shown that
29 both in the region of the medium's motion and in front of the front of the seismic wave,
30 a system of currents and fields perturbing the Earth's EM field appears. In Sect. 2 we
31 show that EM precursors are diffusive in nature, and obtain simple laws that determine
32 the spatial size, characteristic duration and amplitude of the precursor.

33 It is necessary to note that the seismic effects are displayed also at heights of the
34 ionosphere of the Earth. The issues of studying these effects in the ionospheric plasma
35 caused by seismic phenomena are currently attracting the attention of researchers in
36 connection with the importance of this problem not only for "pure" science, but also
37 for ensuring the safe life of the population in seismically dangerous regions and on the

38 planet as a whole. At the same time, an important role is played by the study of seismo-
 39 ionospheric post-effects, for example, to better understand cause-effect relationships in
 40 the system "solid Earth – atmosphere – ionosphere", for allocation of seismically
 41 caused oscillations in the spectrum of ionospheric fluctuations, etc. Sect. 3 presents the
 42 results of a theoretical study of the problem, taking into account weak nonlinearity,
 43 dispersion and the influence of dissipation in 3D geometry.

44 2 Seismo-Electromagnetic Effects

45 Consider a conducting homogeneous medium with a constant coefficient of electrical
 46 conductivity σ located in a uniform magnetic field \mathbf{B}_0 . At time $t=0$, a spherically sym-
 47 metric acoustic longitudinal wave arises. The quasistationary Maxwell equations (at B
 48 $\ll B_0$) have form: $\partial_t \mathbf{B} = D \Delta \mathbf{B} + \text{rot}[\mathbf{v}, \mathbf{B}_0]$, $\mu_0 \text{rot} \mathbf{B} = \sigma(\mathbf{E} + [\mathbf{v}, \mathbf{B}_0])$ where
 49 $\mathbf{v} = v(r, t) \mathbf{e}_r$ is the preset velocity field, $D = (\mu_0 \sigma)^{-1}$ is the magnetic viscosity. We
 50 choose the spherical coordinate system (r, θ, φ) with the reference point at the
 51 center of symmetry. Then only three components of the field, B_r, B_θ, E_φ , which de-
 52 pend on the variables r, θ, t , will be nonzero. Let us study the solutions of the Maxwell
 53 equations for the far zone from the nidus ($r \gg R, r \gg r_d \gg \lambda$) and time $t \gg t^* = D/C_l^2$
 54 (C_l is the longitudinal wave velocity). Analyzing solution near the front of the elastic
 55 wave at $r \sim C_l t$ we can obtain that for the front region ($\varepsilon \geq 0$) it has form [3, 5]

$$56 \quad \begin{aligned} B_r &= -\frac{2B_0 R \lambda G(t, \varepsilon)}{r^2} \left(1 + \frac{\lambda}{r}\right) \cos \theta, & B_\theta &= -\frac{B_0 R G(t, \varepsilon)}{r} \left(1 + \frac{\lambda}{r} + \frac{\lambda^2}{r^2}\right) \sin \theta, \\ E_\varphi &= \frac{B_0 R C_l G(t, \varepsilon)}{r} \left(1 + \frac{\lambda}{r}\right) \sin \theta, & G &= \exp\left(-\frac{\varepsilon}{\lambda}\right) \int_0^t \exp\left(-\frac{t'}{t^*}\right) \partial_t^3 f(t') dt'. \end{aligned} \quad (1)$$

57 where $\lambda = D/C_l$, $t^* = D/C_l^2$, $\varepsilon = r - C_l t - R$ and $f(\xi)$ is the reduced elastic dis-
 58 placement potential. Behind the wave front, i.e. when $\varepsilon < 0$, the solution has form

$$59 \quad \begin{aligned} B_r &= -\frac{2B_0 R \lambda}{r^2 C_l} \left\{ \left(1 + \frac{\lambda}{r}\right) G_1(t, \varepsilon) + \partial_t^2 f\left(-\frac{\varepsilon}{C_l}\right) + \frac{C_l}{\lambda} G_2(\varepsilon, r) \right\} \cos \theta, \\ B_\theta &= -\frac{B_0 R}{r C_l} \left\{ \left(1 + \frac{\lambda}{r} + \frac{\lambda^2}{r^2}\right) G_1(t, \varepsilon) + \left(1 + \frac{\lambda}{r}\right) \partial_t^2 f\left(-\frac{\varepsilon}{C_l}\right) + \frac{C_l}{r} G_2(\varepsilon, r) \right\} \sin \theta, \\ E_\varphi &= \frac{B_0 R C_l}{r} \left\{ \left(1 + \frac{\lambda}{r}\right) \left[\partial_t^2 f\left(-\frac{\varepsilon}{C_l}\right) + G_1(t, \varepsilon) \right] + \frac{C_l}{r} \partial_t f\left(-\frac{\varepsilon}{C_l}\right) \right\} \sin \theta, \end{aligned} \quad (2)$$

$$62 \quad G_1 = \exp\left(-\frac{\varepsilon}{\lambda}\right) \int_{-\varepsilon/C_l}^t \exp\left(-\frac{t'}{t^*}\right) \partial_t^3 f(t') dt', \quad G_2 = \partial_t f\left(-\frac{\varepsilon}{C_l}\right) + \frac{C_l}{r} f\left(-\frac{\varepsilon}{C_l}\right).$$

63 Formulas (2) relate to the seismic zone, and formulas (1) describe the structure of the
 64 EM wave, a precursor. It follows from (1) that in the precursor region the field decreases

65 exponentially with characteristic scale λ . This result can be explained as follows. Since
 66 the precursor has a diffusion character, that the characteristic diffusion propagation ve-
 67 locity $d_t r_d \sim (D/t)^{1/2}$ should be of the order of the velocity of the source of geomag-
 68 netic disturbances. We can find from here the characteristic duration $t^* \sim D/C_l^2$ and
 69 spatial scale $\lambda \sim C_l t^* \sim D/C_l$ of the EM precursor.

70 At long distances ($r \gg R$), the expression for a longitudinal spherical wave is a
 71 combination of two half-waves: compression and rarefaction, and we can obtain [5]

$$72 \quad \frac{B_\theta}{A} = \begin{cases} \exp\left(-\frac{\varepsilon}{\lambda}\right) \left[v_1 - (v_1 + v_2) \exp\left(-\frac{\tau_1}{t_*}\right) + v_2 \exp\left(-\frac{\tau_1 + \tau_2}{t_*}\right) \right], & \varepsilon \geq 0; \\ \exp\left(-\frac{\varepsilon}{\lambda}\right) \left[v_2 \exp\left(-\frac{\tau_1 + \tau_2}{t_*}\right) - (v_1 + v_2) \exp\left(-\frac{\tau_1}{t_*}\right) \right] + v_1, & 0 > \varepsilon \geq -C_l \tau_1; \\ v_2 \left[\exp\left(-\frac{\varepsilon}{\lambda} - \frac{\tau_1 + \tau_2}{t_*}\right) - 1 \right], & -C_l \tau_1 > \varepsilon; \end{cases}$$

$$73 \quad E_\varphi = -B_\theta C_l; \quad A = -B_0 R \sin \theta / (r C_l). \quad (3)$$

74 As for the amplitude of magnetic field of precursor B^* in the far zone, we can obtain
 75 [3]: $B^* = -B_\theta(0) = B_0 R \mu_0^2 \sigma^2 C_l^3 v_1 \tau_1 (\tau_1 + \tau_2) \sin \theta / (2r)$. An analysis of (3) shows that
 76 when passing from the precursor region ($\varepsilon > 0$) to the focal point ($\varepsilon < 0$), B_θ changes its
 77 sign, and the magnitude of the field reaches to its maximum amplitude
 78 $B_{\max} = B_\theta(-C_l \tau_1) = B_0 R \mu_0 \sigma C_l v_2 \tau_2 \sin \theta / r$ at $\varepsilon = -C_l \tau_1$. This represents the ampli-
 79 tude of the main signal that occurs after the arrival of a seismic wave to the observation
 80 point, i.e. the amplitude of the induction seismomagnetic effect. From here, we obtain:
 81 $B^* / B_{\max} = [\mu_0 \sigma C_l^2 v_1 \tau_1 (\tau_1 + \tau_2)] / 2v_2 \tau_2 \sim l / \lambda = (\tau_1 + \tau_2) / t_* \ll 1$ where $l = C_l (\tau_1 + \tau_2)$
 82 is the acoustic wavelength, and λ is the precursor wavelength.

83 The results obtained can be explained as follows. A seismic wave generates external cur-
 84 rents with a density $\mathbf{j}_{cm} = \sigma [\mathbf{v}, \mathbf{B}_0]$. Expressions (1) give the field of the effective mag-
 85 netic moment \mathbf{p}_m taking into account the screening arising due to the skin effect. Assuming
 86 $\sigma \rightarrow 0$, we obtain: $B_r = -2G_3 \lambda^2 r^{-3} \cos \theta$, $B_\theta = -G_3 \lambda^2 r^{-3} \sin \theta$, $G_3 = (B_0 R / C_l) \partial_t^2 f(t)$,
 87 whence it is seen that $\mathbf{p}_m = -4\pi G_3 \lambda^2 \mathbf{B}_0 / (B_0 \mu_0)$. The minus sign here is due to the
 88 diamagnetic effect of a moving conducting medium, therefore, the projections of mag-
 89 netic disturbances at long distances from the epicenter are negative and the EM precu-
 90 sor signal is also negative.

91 3 Manifestation of Seismic Effects in the Ionosphere

92 When considering the acoustic disturbance in the near zone of earthquake, we can ob-
 93 tain the approximate expression for the perturbation of electron density from the conti-
 94 nuity equation as [5]:

$$\begin{aligned}
N'_e(t, z, r, \varphi) &\approx (2\pi R)^{-1} i N_0(z) \cos \alpha \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \Gamma(\omega, z, r) \times \\
&\times [1/2H(z') + i \varepsilon(\omega, R) \sin \chi / c(z')] - \\
&- (2\pi R)^{-1} i N_0(z) \sin \alpha \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \Gamma(\omega, z, r) \times i \varepsilon(\omega, R) \cos \chi / c(z'), \\
\Gamma(\omega, z, R) &= e^{-i\omega t} V_0(\omega) [-R_0(\omega, R) \cos \alpha + R_1(\omega, R) \cos \varphi \cos \alpha] \times \\
&\times \exp \left\{ \int_0^z dz' / 2H(z') + i \int_0^R dR' \varepsilon(\omega, R') / c(R') \right\}
\end{aligned} \tag{4}$$

97 and φ is the angle between the planes $(\mathbf{z}, \mathbf{H}_0)$ and (\mathbf{z}, \mathbf{V}) .

98 To obtain quantitative estimates of the magnitude of perturbations of the electron
99 density in the ionosphere F -layer caused by non-stationary oscillations of the earth's
100 surface as a result of an earthquake, we performed calculations in accordance with for-
101 mula (4) for various spatial scales of the earthquake L for three types of approximation
102 of the displacement velocity of the earth's surface: $V_0(t) = S_1 \beta_1^2 t \exp(-\beta_1 t)$;
103 $V_0(t) = 2S_2 \Theta_1 \beta_2 t \exp \Theta_1$, $\Theta_1 = 1 - \beta_2 t^2$; $V_0(t) = \frac{1}{4} S_3 \Theta_2 \beta_3^2 t \exp \Theta_2$, $\Theta_2 = 2 - \beta_3 t$.

104 The first approximation corresponds to earthquakes, which lead to an increase or
105 decrease of the level of the earth's surface with maximum amplitude S_1 at point $r = 0$.
106 Two other approximations describe earthquakes accompanied by oscillations of the
107 earth's surface with different relaxation times.

108 We have obtained that the ionosphere response is quasiperiodic in nature with oscil-
109 lation periods about 40–80 s, moreover, the amplitude of the response substantially
110 depends both on the temporal nature of the initial disturbance of the earth's surface and
111 on its spatial scale. This can be explained by the filtering properties of the atmosphere
112 with a frequency band from the acoustic cutoff frequency ω_a to the upper frequency,
113 which is defined by the viscosity of the atmosphere. The amplitude of the perturbation
114 decreases and the quasi-period increases with distance.

115 As for a far earthquake zone, that the Rayleigh wave $V_z|_{z=0} = d_r Z(r', t)$,
116 $Z(r', t) = h(t) \exp[-(r')^2 / L^2]$ (here $(r')^2 = \xi^2 + y^2$, $\xi = x - v_R t$, v_R is the velocity
117 of the wave) leads to the formation of a wave going upward with an amplitude growing
118 with height, which is associated with an exponential decrease of density:
119 $\rho_0(z) = \rho_0(0) \exp(-z/H)$. Nonlinear effects begin to manifest themselves at the
120 heights of the ionosphere F -region, when a nonlinear solitary IGW is formed under the
121 action of the going upword wave excited by the surface Rayleigh wave [6]. Taking into
122 account the geometry of the problem and weak nonlinearity for the velocity of neutral
123 particles $u(t, r', z) = V(t, r, z)|_{x=\xi+vt}$ at $\partial_z = 0$, we obtain the BK equation [6, 7]:

$$\partial_t u + \frac{2\gamma - 1}{\gamma^2} u_z \partial_\xi u - \sigma \partial_\xi^2 u + 2 \frac{(\gamma - 2)^2}{\gamma^2} vH \partial_\xi^3 \left[u + \frac{(\gamma - 2)^2}{2\gamma^2} \varepsilon H^2 \partial_\xi^2 u \right] = \frac{v}{2} \int_{-\infty}^{\xi} \partial_y^2 u d\xi \tag{5}$$

125 where $\gamma = C_p / C_v$, $\varepsilon = -v / v_{\min}^{ph}$, v_{\min}^{ph} is the minimum phase velocity of linear oscilla-
126 tions, σ is the viscosity coefficient. Considering the solitary waves traveling at the near-

127 to-horizontal angles, we can obtain the solution of the continuity equation for the elec-
128 tron density N_e in the F -layer [6, 7] as follow:

$$129 \quad N_e(u, t) = N_e(u, t_0) \exp[\mathfrak{I}(u, t)], \quad \mathfrak{I}(u, t) = \int_{t_0}^t g(u, t) dt, \quad (6)$$

$$130 \quad g(u, t) = C - (1/H_i + 1/2H)f(u, t), \quad C = 3a/H_i^2 - \beta(1 - q),$$

$$131 \quad f(u, t) = uc \exp(z/2H)(1 - e^{-vt'}) \sin I \cos I, \quad q = Q/\beta N_e, \quad a = D_\alpha \sin^2 I.$$

132 where $D_0 \exp(z/H_i) = D_\alpha \sin^2 I$, D_α is the ambipolar diffusion coefficient,
133 $\beta = \beta_0 (-Pz/H_i)$ and Q are, respectively, the recombination rate and the ion produc-
134 tion rate; $t' = t - t_0$ where t_0 is the moment of the start of the neutral component's per-
135 turbation; H_i is the scale height for ions. Function u in solution (6) satisfies Eq. (5).

136 The results of integration of Eqs. (5), (6) for typical values of the parameters of the
137 F -layer and the disturbances travelling with the velocities $\sim 200 \text{ ms}^{-1}$, were presented
138 in our papers and book [5-7]. It was shown there that for
139 $N' = \{[N(u, t) - N(0, t)]/N(0, t)\} \times 100\%$ function $N'(u, t)$ has a wave character with
140 an increasing steepness of the leading front like a shock wave. Thus, such TID excited
141 by the Rayleigh wave can be consider as some post-effect of the earthquake.

142 4 Discussion

143 The analysis presented in Sect. 2 shows that an EM precursor appears in front of the
144 seismic wave front in a conducting medium, and the precursor amplitude exponentially
145 decreases with distance, with a characteristic scale $\lambda = (\mu_0 \sigma C_l)^{-1}$. For the upper layer
146 of sedimentary rocks, $\lambda \sim 100 \text{ km}$, therefore, the precursor can be ahead of the elastic
147 wave by no more than a few seconds. The precursor amplitude increases with time up
148 to the moment of arrival of the seismic wave. At distances of the order of tens of kilo-
149 meters from the epicenter, its amplitude can reach values from several pT to nT for
150 magnetic disturbances and from several nV/m to $\mu\text{V/m}$ for electric ones, depending on
151 the medium conductivity and seismic wave parameters. With increasing conductivity,
152 the precursor amplitude increases, and its characteristic size λ decreases.

153 As for the seismic effects in the ionosphere in near and far zones from the nidus of
154 earthquake (Sect. 3), note that the study of the multidimensional case, taking into ac-
155 count all significant factors (weak nonlinearity, dispersion and dissipation), allows us
156 to obtain more accurate results for both the near and far zones of epicenter. Thus, for a
157 few types of approximation of the displacement velocity of the earth's surface it was
158 shown that in the near zone the ionosphere response is quasiperiodic with oscillation
159 periods about 40–80 s, and its amplitude depends on temporal and spatial scales of the
160 initial disturbance of the earth's surface. At this, the amplitude decreases and the quasi-
161 period increases with distance. On big distances it is necessary to take into account that
162 spatial dispersion leads to damping of the oscillations of the acoustic branch with prop-
163 agation and a shift of the spectral maximum to a lower-frequency region is observed.
164 In this case, the surface Rayleigh wave excites the perturbation of the neutral compo-
165 nent of the atmosphere in form of a solitary IGW which is a source of the solitary TID

166 at heights of the *F*-layer of the ionosphere. Nonlinear effects lead to increasing steep-
 167 ness of the TID leading front, and dissipation leads to the exponential decay of the
 168 perturbation with decreasing its amplitude.

169 5 Conclusions

170 In conclusion, we have presented the results of study of displaying of seismic activity in
 171 variations of both the EM field and the basic ionospheric characteristics. We have showed
 172 that considering of the 3D case with a due account of the effects of weak nonlinearity,
 173 dispersion and dissipation in medium enables to obtain more accurate results for both the
 174 near and far zones of the earthquake epicenter. In study of the seismic response in the EM
 175 field, it was confirmed that a precursor arises ahead the front of seismic wave, and it was
 176 shown that its amplitude depends on the medium conductivity and the parameters of the
 177 wave. Regarding the response at ionospheric heights, the seismo-ionospheric post-effects
 178 were studied, that is of great interest, in particular, for a better understanding of relation-
 179 ships in the system "solid Earth – atmosphere – ionosphere" and for identification of seis-
 180 mically caused oscillations in the spectrum of the ionospheric fluctuations, etc. The effect
 181 of the acoustic impulse caused by the Rayleigh wave on the ionosphere's neutral compo-
 182 nent near the epicenter was also considered, and formation of the solitary IGWs and the
 183 TIDs, which are caused by it, at heights of the *F*-layer in the far zone was studied.

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