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Multidimensional gravity with higher derivatives and inflation

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ABSTRACT

We elaborate on the inflationary model starting from multidimensional Lagrangian and gravity with second-order curvature terms. The effective scalar field is related to the Ricci scalar of extra dimensions. It is shown that the Kretschmann and the Ricci tensor square terms dominate during inflation. The observable values of the spectral index and the tensor-to-scalar ratio are obtained for specific values of the model parameters.

1. Introduction

It is usually assumed that our Universe was nucleated at the Planck energies and evolved by expanding and cooling to the present condition. The inflationary stage is characterized by sub-Planckian energy density and looks unavoidable. Spontaneous creation of the Universe with the inflationary regime has been elaborated in [1]. Models describing the inflation have been elaborated using many different ingredients, like supersymmetry [2] and attracting the inflationary idea for an explanation of other cosmological problems like baryogenesis [3] and primordial black holes [4], [5]. For review, see [6,7] and references therein. On the other hand, as the energy scale is high enough some effects related to a quantum regime may be manifested and be responsible for the inflationary regime. Two typical elements from quantum gravity may be relevant in the construction of inflationary models: non-linear geometrical extensions of General Relativity and extra dimensions. Moreover, the quantization of gravity asks for non-linear geometric extension of the Einstein-Hilbert action. The first and most successful formulation of the inflationary model, the Starobinsky model [8], considers non-linear geometric terms belonging to the $f(R)$ class of theories.

The gravity with higher derivatives is widely used in modern research despite the internal problems inherent in this approach [9], [10]. Attempts to avoid the Ostrogradsky instabilities are made [11] and extensions of the Einstein-Hilbert action attract much attention. The $f(R)$-gravity is one of the simplest extension of the Einstein-Hilbert gravity and it is free of Ostrogradsky instabilities. Reviews [12], [13] contain description of the $f(R)$-theories including extension to the Gauss-Bonnet gravity. Examples of research with a specific form of the function $f(R)$ can be found in [14], [15]. Most of the research assumes positive curvature of extra space metric.

Another widespread idea, the extra-dimensional world is considered as the necessary element for a fundamental complete theory. The paradigm of the extra dimensions is also attracted for an explanation of the cosmological evolution [16]. The simplest way to explain their invisibility is an imposing a compact extra space with the size smaller than 10^{-18} cm.

Our research aims to unify two necessary elements of a future theory - the (compact) extra dimensions and the gravity with higher derivatives to build a model of inflation. No matter fields are assumed from the beginning. It is shown that the Kretschmann and the Ricci tensor square terms dominate during inflation and hence play a significant role in our research.

The inflation gives information which should be reproduced by our scenario. More definitely, it is assumed that the effective potential should provide us with the reheating stage and appropriate values of the scalar spectral index n_s and the scalar/tensorial relative amplitude r. In the present paper, we investigate the conditions to have a successful inflationary model from a non-linear theory of gravity in higher dimensions.

Throughout this paper we use the conventions for the curvature tensor $R^D_{ABC} = \partial_C \Gamma^D_{AB} - \partial_B \Gamma^D_{AC} + \Gamma^D_{EC}\Gamma^E_{BA} - \Gamma^D_{EB}\Gamma^E_{AC}$ and for the Ricci tensor $R_{MN} = R^F_{MFN}$.

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In this paper, we express all numbers in units $m_D = 1$ with the effective Planck mass m_4 defined by (13). After the inflation is finished and the scalar field has been settled in the minimum of its potential one should restore more physical units valid for the Jordan frame, i.e. the Planck units using the relation (8). For chosen parameter values (40) the relation

$$M_{Pl} = \sqrt{V_n e^{n\beta_m} f'(\phi_m)} m_D \tag{43}$$

can be used to obtain the value of the D-dim Planck mass. For 2-dim extra space ($n = 2, \quad V_2 = 4\pi$)

$$M_{Pl} = \sqrt{V_2 e^{2\beta_m} f'(\phi_m)} m_D = \sqrt{8\pi \left(2a + \frac{1}{\phi_m}\right)} m_D \sim 10 m_D \tag{44}$$

that means that the D-dim Planck mass m_D is in the order of magnitude smaller than the Planck mass in the Jordan frame.

5. Conclusion

The inflationary model was studied in the framework of nonlinear multidimensional gravity without the matter fields. The Ricci scalar of the extra space plays the role of inflation. Parameters of the model are adjusted to satisfy observational values for the spectral index and the tensor-to-scalar ratio. The model has a large parameter ($c_K \sim 10^5$) which is a common feature of the inflationary models. Our study points that the Kretschmann and the Ricci tensor square terms dominate during inflation so that our model differs essentially from the Starobinsky R^2 model [8] leading, however, to similar results. According to the observational data, inflationary models with a single scalar field and simple form of potential are not very perspective. Our model leads to a complex form for the effective potential and kinetic factor that give promising results. We have given an example where the model studied here is in agreement with the observational constraint. A more complete analysis of the space parameter is, of course, necessary.

The model developed here is the representative of a set of models that differ in the number of extra dimensions, the form of the function $f(R)$ and other invariants. The dependence of the full Lagrangian on the different parameters is very complex, as it can be seen from Eq. (7) and subsequent expressions, so that a detailed analysis is rather cumbersome. For this reason, such an analysis is postponed for future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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