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Evolution of sub-spaces at high and low energies

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Abstract The evolution of sub-spaces in the framework of gravity with higher derivatives is studied. Numerical solutions to exact differential equations are found. It is shown that the initial conditions play crucial role in the space dynamic. Appropriate metrics describing an expanding and a stationary sub-space shed light on the well-known question: why our 3-dim space is large but an extra space is small and stable (if exists)? It is assumed that the values of parameters at high energies strongly depend on uncontrolled quantum corrections and, hence, are not equal to their values at low energies. Therefore, there is no way to trace solutions throughout the energy range, and we restrict ourselves to the sub-Planckian and the inflationary energies.

1 Introduction

The origin of our Universe remains as an unresolved problem up to now. It is usually assumed that its nucleation is related to the quantum processes at high energies [1–4]. The probability of its creation remains unclear in spite of wide discussion, see e.g. [5–7]. Here we are interested in the subsequent classical evolution of the metrics rather than a calculation of this probability. It is assumed that manifolds can be described by specific metrics after their nucleation. After nucleation, these manifolds evolve classically forming a set of asymptotic manifolds, one of which could be our Universe.

The complexity of the problem is greatly aggravated by two factors. First, the metric evolution should lead to the formation of our Universe with the strong fine-tuning of the observational parameters [8,9]. Second, the inclusion of extra dimensions is of particular interest because the idea of extra space is widely used in modern research. They shed light to such issues as the grand unification [10,11], neutrino mass [12], the cosmological constant problem [13–15] and so on. In this regard, the immediately aroused question is: why specific number of dimensions are asymptotically compact and stable while others expand [16–18]? Which specific property of subspace leads to its quick growth? Sometimes one of the subspaces is assumed to be FRW space by definition [19]. There are many attempts to clarify the problem, mostly related to introduction of fields other than gravity. It may be a scalar field [16,20] (most used case) and gauge fields [21] for example. A static solution can be obtained using the Casimir effect [22,23] or form fields [24]. Another possibility was discussed in [25,26]: it was shown that if the scale factor $a(t)$ of our 3D space is much larger than the growing scale factor $b(t)$ of the extra dimensions, a contradiction with observations can be avoided.

In our previous article [27] we studied evolution of manifolds after their creation on the basis of pure gravitational Lagrangian with higher derivatives. It was shown analytically and confirmed numerically that an asymptotic growth of the manifolds depends weakly on initial conditions. We have shown that the initial conditions can be a reason of nontrivial solutions (funnels) and studied their properties. A number of final states of metric describing our Universe is quite poor if we limit ourselves with a maximally symmetric extra space and the $f(R)$ gravity.

In this article, we continue to study the Universe evolution at the sub-Planckian scale. The space V_D is assumed to be the direct product $T \times V_3 \times W_3$ of the time and the two maximally symmetric manifolds with positive curvature. Both sub-spaces are born with the size of the order of the Planck scale or more. Most of the resulting sub-spaces are characterized by initial metrics, which lead to the growth of both 3-dimensional sub-spaces, which clearly contradicts the observations. We have found a set of metrics that could lead to the observable space-time metric.

The action used should not contradict the observations at the low and intermediate energies. More definitely, the Lagrangian parameters at low energies should be chosen in such a way to supply (almost) Minkowski space for the mod-

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realized in the Nature the Einstein frame or the Jordan one. In the latter case, we have to turn to Eq. (23) after the evolution of the extra space is finished and the Ricci scalar $R_3 = \phi_0$. The expression for the 4-dim Planck mass can be obtained by equating the value $M_P^2/2$ to the multiplier to R_4

$$M_P^2 = v_3 e^{3\beta_0} f'(\phi_0) m_D^2.$$

Here expression (28) at the potential minimum $\phi = \phi_0$ is taken into account and the parameter m_D is restored. For chosen parameter values $M_P \simeq 700 m_D$ and previous estimations based on the Einstein frame remains the same.

We conclude that the compact space volume could remain small enough during the whole period of its evolution – from its nucleation at the (sub-)Planckian energies up to the modern epoch.

5 Quantum fluctuations and stability of extra space

In this section, we shortly discuss the role of quantum fluctuations on the stability of the extra space metrics discussed above. We start with the low energy scale M_{low} where the present horizon is formed. The quantum fluctuations of the scalar field ϕ have been intensively studied [41,42]. The common conclusion is that in spite of their smallness they are the reason of the large scale structure formation in our Universe. The amplitude of the fluctuations during the slow roll inflation is of the order of the $\delta\phi \sim H/2\pi \sim 10^{-7} M_P$. They are also very small in the m_D units: $\delta\phi \sim 10^{-6}$.

In this research, the Ricci scalar $R_3(\beta)$ plays the role of the scalar field ϕ – the inflaton – with the potential (27). Estimations made above indicate that the inflationary stage is performed near the bottom of the potential far from the potential extrema, see Fig. 6. Therefore we may not worry about the influence of the fluctuations on the final state of inflaton. If the initial value of the inflaton is to the right of the maximum, it will inevitably move to nonzero minimum. This indicates the stability of the metric evolution $\beta(t) = -\frac{1}{2}\ln[R_3(t)/6] = -\frac{1}{2}\ln[\phi(t)/6] \to -\frac{1}{2}\ln[\phi_{end}/6]$ with respect to the radial quantum fluctuations at the inflationary scale of energy.

The problem arises when we shift the scale up to the sub-Planckian energies where the quantum fluctuations $\delta\beta$ are of the order of the unity. Their role is twofold. If the region of stability is small, the quantum fluctuations could easily break the stationary behaviour of the extra space. The latter starts expanding or shrinking so that the probability of staying in a stable region is very small. On the other side, suppose that there exists the set of parameters a, k, c leading to quasi stable classical solutions with slowly expanding sub-spaces W_3. Quantum fluctuations are able to turn its metric back to a stationary regime in some causally connected domains of the large sub-space V_3. Such domains could survive up to the beginning of the inflation.

6 Conclusion

The appearance of manifolds with different metrics as a result of quantum effects at high energies is a well known paradigm. After their creation, some manifolds evolve classically. The originated metrics serve as the initial conditions for their subsequent classical evolution. The measure of any metric originated from space-time foam is assumed nonzero though uncertain due to the absence of the Theory of Quantum Gravity.

The analysis performed in this paper indicates that there are several regimes of sub-spaces evolution at sub-Plankian energies. There are regimes characterized by the expansion of both sub-spaces at equal rates as well as at different rates depending on the Lagrangian parameters. We also have shown that some sub-spaces come back to the space-time foam.

The observable fact is that the only one 3-dim sub-space is large. It is those space where the modern physical processes are performed. Therefore, the regime characterized by only one growing sub-space is of most interest. We have found that such a regime is realized at the highest energies for specific values of the Lagrangian parameters $a(m_D)$, $k(m_D)$, $c(m_D)$ and the specific initial metrics.

The parameter values of the Lagrangian $a(M_{infl})$, $k(M_{infl})$, $c(M_{infl})$ at the inflationary scale M_{infl} were also discussed on the basis of chaotic inflation. They do not coincide with those at high energies m_D due to uncontrolled quantum corrections at the sub-Planckian energy scale.

Shortly, the general picture is as follows. Sub-spaces are nucleated with different initial metrics. There is a class of multidimensional models with the higher derivatives for which some of the sub-spaces form pairs evolving classically in proper manner - one of the sub-spaces expands while the other remains constant.

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