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Dynamical systems perspective of cosmological finite-time singularities in f (R) gravity and interacting multifluid cosmology

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Abstract

© 2018 American Physical Society. In this work we shall investigate the occurrence of future cosmological finite-time singularities in the dynamical system corresponding to two cosmological theories, namely that of vacuum f(R) gravity and that of three fluids. As we shall make clear, a finite-time cosmological singularity may be entirely different from a finite-time singularity of a dynamical system, since the latter mainly depends on the behavior of the dynamical system variables. The vacuum f(R) gravity is an example for which the variables we will choose to quantify the phase space dynamics, do not necessarily blow up near a cosmological singularity. After appropriately choosing the variables, we shall investigate the behavior of the corresponding dynamical system near some types of cosmological finite-time singularities, for some limiting cases in which we can produce analytic solutions for the dynamical variables. The most interesting case from both a mathematical and physical point of view, is the big rip case, and particularly in the limiting case of a very strong singularity. The physically appealing outcome is that the resulting nonautonomous dynamical system is attracted asymptotically to an accelerating attractor solution, with equation-of-state parameter weff=-1. Our analytic results, show that an extremely strong big rip singularity in vacuum f(R) gravity theories is always related to an accelerating solution, or tends to acceleration. The converse statement though may not be true. We also perform the same analysis for the Type IV finite-time singularity, and we investigate the behavior of the dynamical system near the Type IV singularity, in the case that the singularity is extremely soft, in which case we are able to produce analytic expressions for the dynamical solutions. Also we briefly discuss how the removal of the finite-time singularity may be achieved by the addition of an R2 term in the f(R) gravity action. The second cosmology we shall study is a multifluid cosmology, consisting of three fluids: the interacting dark matter and dark energy fluids, and the baryonic fluid. By appropriately choosing the variables, we will show that the dynamical system can become an autonomous polynomial dynamical system, in which case, by using a dominant balance analysis, we shall investigate the occurrence of finite-time singularities in this system. We also study numerically and analytically, in some detail, the phase space of the dynamical system for some specific forms of the dark energy-dark matter interaction term.

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