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Kramers-Kronig relations and causality conditions for graphene in the framework of the Dirac model

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Abstract

 We analyze the concept of causality for the conductivity of graphene described by the Dirac model. It is recalled that the condition of causality leads to the analyticity of conductivity in the upper half-plane of complex frequencies and to the standard symmetry properties for its real and imaginary parts. This results in the Kramers-Kronig relations, which explicit form depends on whether the conductivity has no pole at zero frequency (as in the case of zero temperature when the band gap of graphene is larger than twice the chemical potential) or it has a pole (as in all other cases, specifically, at nonzero temperature). Through the direct analytic calculation it is shown that the real and imaginary parts of graphene conductivity, found recently on the basis of first principles of thermal quantum field theory using the polarization tensor in (2+1)-dimensional space-time, satisfy the Kramers-Kronig relations precisely. In so doing, the values of two integrals in the commonly used tables, which are also important for a wider area of dispersion relations in quantum field theory and elementary particle physics, are corrected. The obtained results are not of only fundamental theoretical character, but can be used as a guideline in testing the validity of different phenomenological approaches and for the interpretation of experimental data.

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