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THE DERIVATION OF THE FORMULA FOR ARCH DEFLECTION BY THE METHOD OF DOUBLE INDUCTION IN THE MAPLE SYSTEM

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The truss with the cross-shaped lattice of the U-shape has two fixed supports. The analytical dependence of the deflection of the truss is derived from the number of panels in the middle part and the number of panels in the side parts. Operators of the Maple system were used to compile and solve recurrent equations.

Keywords: truss, deflection, Maple, induction.

The method of induction on one parameter using the computer mathematics system Maple [1] is widely used for obtaining analytical dependencies in the problems of the deflection of rod systems. In [4, 3, 4, 5, 6, 7] polynomial solutions for the number of panels were obtained for flat trusses, in [8, 9, 10, 11] — for flat lattice designs. Spatial trusses from this point of view were studied in [12, 13, 14]. In this paper we consider the construction of natural images that includes two natural parameters that regulate its dimensions. Hinged core structure (Fig. 1) is a statically determinate truss. In General, the vertical lateral parts of the m panels and the horizontal middle part — $2n$ panels. The total number of cores in the truss, $N = 8(n + m + 1)$ number of nodes $4(n + m + 1)$. Derive formula for deflection of this structure under the action of a concentrated force P at Midspan.

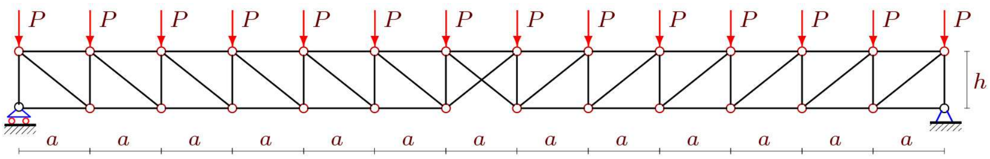

Fig. 1. Truss, $n = 6$, $m = 2$

To determine the stresses in the bars will use the program [1] written in systems of symbol mathematics Maple. The program incorporates a method of cutting of knots. In the original these programs are introduced the coordinates of the nodes and vectors that specify the order of connection of nodes and cores in the same way as in discrete mathematics a graph is defined by edges and vertices. Next, create a matrix $\boldsymbol{G}$ of the system of equilibrium equations of nodes, consisting of the guides of the cosines of the rods, which in turn are calculated using the coordinates of the nodes. Two fixed supports are modeled by rods whose lengths can be chosen arbitrarily, since the supporting rods are assumed to be rigid. It should be noted that a feature of the proposed design is that the traditional calculation scheme of trusses are not held. The calculation usually begins with the determination of reactions of supports. However, to determine the four reactions of supports of the three equilibrium equations of the whole structure in General is impossible. The method of partitioning in part by analogy with the solution of problems in compound structures, where there is the same problem here also out. The reaction in this case you can define together with the efforts of all the rods from the solution of the global system of equilibrium equations for all nodes. The system of equations ***GS=B*** is solved analytically using the built-in operators of the system Maple.

The method of inverse matrix is used, the more that it is implemented in Maple is very simple. The inverse matrix ***G1*** is calculated by simple division: ***G1:=1/G***. the Vector of the right part of the system of equations associated with external loads. In odd-numbered items are horizontal loads (in this problem they are not), even vertical ***B*[2*(*n*+*m*)]:=1**. The solution is obtained by multiplying the inverse matrix by the vector of loads **S1:=G1.B.** To determine the deflection using the formula of Maxwell - Mohr $\Delta = P\sum_{i=1}^{N-4} S_i^2 l_i/(EF)$ to calculate the deflection. Here *EF* is the stiffness of the rods, S_i – stress in the rods from the action of a unit vertical force applied at the middle node, l_i is the length of the rods. Four rigid support rods are not included in the sum. We consider the case of a fixed number of panels in the lateral parts of the truss, $m = 2k = 4$. Consistent calculation of trusses with *n*=1,2,... 12 showed that the equation for the deflection has the form

$$EF\Delta = P(A_n a^3 + C_n c^3 + H_n h^3)/(2h^2), \tag{1}$$

where $c = \sqrt{h^2 + a^2}$. The method of induction with the involvement of operators **rgf_findrecur** and **rsolve** package genfunc we obtain the total members of the sequence of the coefficients:

$$A_n = [4n^3 - 15n^2 + 27(-1)^n n^2 + 83n - 69(-1)^n n]/6 - 8(-1)^n + 9,$$

$$C_n = 11 - 10(-1)^n + n, \quad H_n = (53 - 43(-1)^n)/2.$$

The most difficult for inductive inference was the coefficient of a^3. The equation for this coefficient has the form

$$A_n = A_{n-1} + 3A_{n-2} - 3A_{n-3} - 3A_{n-4} + 3A_{n-5} + A_{n-6} - A_{n-7}.$$

Figure 2 shows the curves of the dependences for a fixed span of *L*=2*an*=100 m. Indicated dimensionless deflection $\boldsymbol{\Delta' = \Delta EF/(PL)}$. Noticeable surges trough and a marked dependence on the height *h*.

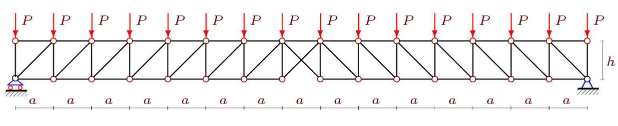

Fig. 2. The deflection- number of panels dependence, $m = 4$

The reaction of supports are also obtained in a symbolic form by induction from the solution of the General system of equilibrium equations for all nodes. Horizontal reaction depends on the parity of *n*: $X_A = Pa((-1)^n - 1)/(4h)$, the vertical is constant $Y_A = P/2$. In addition, as shown by the bill, these decisions do not depend on the number *m* of the side panels.

The resulting solution can be generalized to an arbitrary number of panels in the side (supporting) parts of the truss (Fig. 3). As a result of induction on the parameter

$m = 2k$, we have the following formulas

$$A_n = (4n^3 + [(-1)^n(12k+3) - (12k-9)]n^2 +$$
$$+[24k^2 - 12k + 11 - (24k^2 - 12k - 3)(-1)^n]n \; + 12k^2(1 - (-1)^n) + 6)/6,$$
$$H_n = (16k^3 + 14k + 3 - (2k - 3 + 16k^3)(-1)^n)/3,$$
$$C_n = 2k^2 + k + 1 + n - (2k^2 + k)(-1)^n.$$

A review of papers on the topic of analytical calculations in the calculation of flat trusses is contained in [15].

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ВЫВОД ФОРМУЛЫ ДЛЯ ПРОГИБА АРОЧНОЙ ФЕРМЫ МЕТОДОМ ДВОЙНОЙ ИНДУКЦИИ В СИСТЕМЕ MAPLE

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Ферма с крестообразной решеткой П-образной формы имеет две неподвижные опоры. Дан вывод аналитической зависимости прогиба фермы от числа панелей в средней части и числа панелей в боковых частях. Использованы операторы для составления и решения рекуррентных уравнений системы Maple.

Ключевые слова: ферма, прогиб, Maple, индукция.

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INDUCTIVE STUDY OF THE DEFLECTION OF THE COMPOSITE TRUSS LOADED ON THE UPPER BELT

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Two schemes of a flat statically determinate truss beam type, differing in the direction of the braces, are considered. A mathematical model of the deformation of a structure is constructed under the action of a load uniformly distributed over the upper belt. Formulas for deflection are derived depending on the number of panels.

Keywords: truss, deflection, Maple, induction.

The task is to find an analytical dependence of the deflection of statically determinate symmetric girders on number of spans. To determine the forces in the bars will use the program [1] written in the language of symbolic mathematics Maple. The deflection determined by the formula of Maxwell – Mohr

$$\Delta = \sum_{i=1}^{m-3} S_i N_i l_i / (EF),$$

where the following designations are used: S_i — the forces in the rods of the truss from the action of external loads distributed on the upper zone, N_i the forces in the rods from the action of a single force applied to the node's neighbor to the middle of the span l_i– length of rods, $m = 8(n+1)$ is the number of rods along with three support rods. Forces in the three rigid support members to the amount are not included.

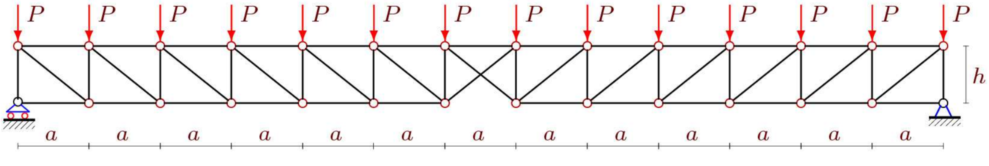

Fig. 1. Truss, downward bracings, $n = 6$

Consistently receiving the solutions of trusses with different number of panels ($n = 1, .., 10$), identify the pattern of formation of the coefficients in the formula for deflection.