On infinite direct sums of lifting modules

Koşan M., Quynh T.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© 2017 World Scientific Publishing Company. The aim of the present article is to investigate the structure of rings $R$ satisfying the condition: for any family $\{S_i|i \in \mathbb{N}\}$ of simple right $R$-modules, every essential extension of $\otimes_i \in NE(S_i)$ is a direct sum of lifting modules, where $E(-)$ denotes the injective hull. We show that every essential extension of $\otimes_i \in NE(S_i)$ is a direct sum of lifting modules if and only if $R$ is right Noetherian and $E(S)$ is hollow. Assume that $M$ is an injective right $R$-module with essential socle. We also prove that if every essential extension of $M(\in \mathbb{N})$ is a direct sum of lifting modules, then $M$ is $\Sigma$-injective. As a consequence of this observation, we show that $R$ is a right $V$-ring and every essential extension of $S(\in \mathbb{N})$ is a direct sum of lifting modules for all simple modules $S$ if and only if $R$ is a right $\Sigma$-$V$-ring.

http://dx.doi.org/10.1142/S1793557117500498

Keywords

H-ring, Hollow module, Lifting module, Noetherian ring, QF-ring, $\Sigma$ -injective module, $\Sigma$ -lifting module, $\Sigma$ -V-ring

References