A.M. Bikchentaev
On convexity and compactness of operator "intervals" on Hilbert space

We consider a von Neumann algebra M acting on a Hilbert space H. For a positive operator X in M we define the operator "intervals" $I_X = \{Y = Y^* \in M : -X \leq Y \leq X\}$ and $L_X = \{Y \in M : |Y| \leq X\}$, where $|Y| = \sqrt{Y^*Y}$. The properties of this operator "intervals" are investigated. We prove that a von Neumann algebra M is Abelian if and only if L_X is convex for all X in M. We then show for $M = B(H)$, the algebra of all linear bounded operators on H, that the operator "interval" I_X is compact if and only if an operator X is compact.