

**ANALYSIS OF LOWER SECONDARY MATHEMATICS TEACHERS' CONTENT  
KNOWLEDGE  
IN USA AND RUSSIA**

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*Abstract.* The study analyzed differences in 8<sup>th</sup> grade students' performance on mathematics portion of TIMSS test through the lens of teacher knowledge. The sample of this study consisted of lower secondary mathematics teachers from US (grades 6-9, N=102) and Russia (grades 5-9, N=97). The instrument was designed to assess teacher content knowledge based on cognitive domains of knowing, applying, and reasoning, as well as addressing lower secondary mathematics topics of Number, Algebra, Geometry, Data and Chance. The study main results suggest that student performance on international tests could be explained by teacher knowledge as well as inform the field on priorities placed on lower secondary mathematics teachers' knowledge in USA and Russia by content and cognitive domains.

*Keywords:* Teacher knowledge, Student performance, Lower secondary school mathematics, TIMSS.

### **Objective**

Motivation for the study is based on the 8<sup>th</sup> grade mathematics portion of the TIMSS-2011 results (Mullis et al. 2012). The assessment used in TIMSS is broken up by content and cognitive domains. The content domains composed of the following topics: Number, Algebra, Geometry, and Data and Chance. The cognitive domain consists of Knowing, Applying, and Reasoning. Analyzing the TIMSS-2011 results at the 8<sup>th</sup> grade in Mathematics, we identified a difference in the US and Russian students' performance on this examination. Overall, average score of Russian students in content domain is 539 and the US students – 509. Russian students also outperformed the US students in each cognitive domain: Knowing – 548 and 519 accordingly, Applying – 538 and 503, and Reasoning – 531 and 503. The TIMSS data triggered a question – could the difference in student performance be explained by Russian and the US teacher performance on a similar test. The question lead us to analyze the US and Russian 8<sup>th</sup> grade students' TIMSS performance in mathematics through the lens of lower secondary teachers' content knowledge in content and cognitive domains. The following research questions guided this study: what parallels if any exist between the US and Russian 8<sup>th</sup> grade students' TIMSS performance in mathematics and the US and Russian lower secondary mathematics teachers' content knowledge? And to what extent the US and Russian lower secondary mathematics teachers' knowledge differ by content and cognitive domains?

### **Perspectives**

Conducting cross-national studies allow comparing, sharing, and learning about issues in an international contexts (Robitaille & Travers, 1992). Cross-national studies also help researchers understand in a more explicit way on their own context, teaching practice, knowledge, and get insights of better choices in constructing the teaching and learning process (Stigler and Perry, 1988). During the last decade, number of cross-national studies on teacher education is increasing in order to understand differences in student performance on international tests such as TIMSS, PISA (Wang & Lin, 2005). Scholars have address these differences focusing on characteristics such as teachers' perceptions of effective mathematics teaching (Cai & Wang, 2009; Hemmi & Ryve, 2015), attitudes and beliefs of mathematics pre-service teachers (Wagner, Lee, & Ozgun-Koca, 1999), teacher knowledge (TEDS-M, 2011; Author, 2015), among others.

Most of the prior studies focused on affective domain of mathematics instruction. Cai, Ding and Wang (2013) conducted a cross-national study to examine US and Chinese in-service teachers' (n=36) view about the meaning of instructional coherence. The study found that instructional coherence is highly

related to discourse. In regards of teacher perceptions of effective mathematics teaching, Hemmi and Ryve (2014) conducted a cross-national study of teacher-educators' perception of effective mathematics teaching in Sweden (n=8) and Finland (n=5). The study found that Swedish teacher educators conceptualize effective teaching as interactions with individual children, building on students' ideas and using mathematics from emerging situations. Finnish teacher educators consider effective teaching in providing clear presentation of mathematics, routines and homework. Some studies focused on cognitive domain of mathematics instruction. For example, Andrews (2008) compared middle school teachers' (n=16) conceptualization and presentation of mathematics in four countries: England, Belgium, Hungary and Spain. The Flemish teachers present a moderate cognitive complexity supported by a moderately high didactic coherence while English teachers present a low cognitive complexity and barely moderate didactic coherence. The Hungarian teachers presented a high cognitive complexity supported by high levels of didactic coherence in contrast with the Spanish teachers comprising low cognitive complexity allied to low didactic coherence.

Few cross-national studies focused on teacher knowledge. Large-scale study conducted by University of Michigan examined mathematical content and pedagogical content knowledge of in-service teachers from 17 countries including USA and Russia (Tatto & Senk, 2011; Tatto, 2013). The development of the items for TEDS-M study was informed by MT21 knowing mathematics for teaching (Ferrini-Mundy, Floden, McCrory, Burril, & Sandow, 2005) and learning mathematics for teaching frameworks (Hill, Ball, and Schilling, 2008). The nature of mathematics teacher knowledge, conceptual representation and curriculum materials were examined by Ma (1999) to explain differences in students' performance in the U.S. and China. An, Kulm, and Wu (2004) studied the PCK of middle school teachers (n=61) in the U.S. and China. They found that mathematical PCK differs since Chinese teachers emphasize developing procedural and conceptual knowledge through traditional teaching practices while their counterparts in the U.S. focus on promoting creativity and inquiry through activities designed to develop student's understanding of mathematical concepts. Sorto et al. (2009) administered surveys that measured teachers' content knowledge (n=385) in Costa Rica and Panama and found that teachers in both countries focus more on knowing rules and procedures than on making connections and reasoning.

The literature review indicates that there is a need to conduct more cross-national studies on in-service teachers' knowledge and its potential impact on students' learning and achievement in mathematics.

### **Methodology**

The proposed study is based on the assessment framework used by TIMSS (Mullis et al. 2012). In this section we will describe the study participants, the instrument as well as data collection and data analysis procedures.

**Participants.** The sample of this study consisted of lower secondary mathematics teachers from US (grades 6-9, N=102) and Russia (grades 5-9, N=97). The US teacher-participants were selected from urban public middle schools in the Southwestern part of the country. Teacher sample demographic information was self-reported by participating teachers. In terms of gender distribution, 55% of teacher participants were females and 45% - males. Most of the US participants (64%) had 1-5 years of teaching experience. Additionally, 62% of the teacher sample received their teaching certificate through traditional teacher preparation programs and 38% of participating teachers were certified through alternative programs. The Russian teacher-participants were selected from urban public secondary schools in the Volga region. Russian participating teachers had attained a secondary mathematics teacher preparation Specialist's degree<sup>1</sup>, which allowed them to teach in secondary schools (grades 5-11). Majority of participating teachers were females (89%). The sample was composed by 78% of teachers who have more than 10 years of teaching experience.

**Instrument.** The instrument used in this study was the Teacher Content Knowledge Survey. It was designed to assess teacher content knowledge based on the three cognitive domains: Knowing, Applying, and Reasoning. The TCKS survey consisted of 33 multiple choice-items topics addressing main

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<sup>1</sup> In Russia, the secondary school consists of lower and upper levels: the lower secondary school includes grades from 5 to 9, and grades 10-11 are part of the upper secondary school.

objectives of lower secondary mathematics curriculum: Number, Algebra, Geometry, Data and Chance. The instrument was field-tested. The alpha coefficient technique (Cronbach, 1951) was utilized to evaluate the reliability of the teacher content knowledge survey. “The value of the coefficient of .839 suggests that the items comprising the TCKS are internally consistent” (Author, 2011).

**Data Collection.** This study implemented data collection procedures at two different levels: (a) teacher and (b) student level. At the teacher level data - measurement of teachers' knowledge was conducted using the TCKS instrument. Each teacher was given 90 min to complete the survey and they were allowed to use graphic calculators during the survey. Along with teachers' scores on the TCKS, teachers' demographic information such as: gender and ethnicity, years of teaching experiences, as well as other proxies for teacher content knowledge (i.e., mathematics coursework) were also collected. Secondary level student data was obtained through TIMSS-2011 study (Mullis et al. 2012).

**Data Analysis.** In correspondence with the research questions, data analysis was performed using the following two major statistical techniques: (a) correlation analysis using standard ordinary least square (OLS) method: the selection of this parametric technique was determined based on the key research question of the study (parallels between teacher knowledge and student achievement), nature of the interval type of data used for teacher knowledge and student performance scores; and (b) non-parametric techniques (chi-square test of goodness of fit) was selected to measure the variance between independent groups of the same (not normal) distribution with arbitrary sample sizes of each group. The selection of this test was also based on the ordinal (ranked) nature of data for content and cognitive domains of teacher knowledge and student performance.

## Results and Conclusions

In this section, we first analyze teacher knowledge data by content domain, then we analyze teacher data by cognitive domain, and finally we analyze parallels between student and teacher performance within and between countries.

The results reported on teacher content knowledge show that the US teachers' highest mean score was obtained on Number domain – 623 and lowest on Geometry domain - 514 ( see Table 1) while Russian teachers' highest mean score was obtained on Algebra domain – 728 and lowest on Data and Chance domain – 387 (see Table 2). Moreover, we found that the US teachers' highest mean score was obtained, as expected, on Knowing domain – 734 and lowest on Reasoning domain - 495 (see Table 3). Russian teachers' highest mean score was obtained, as expected, on Knowing domain – 760 and lowest, unexpectedly, on Applying domain - 504 (see Table 4).

Placed on the same scale, US teachers' and students' performance on Content Domain closely parallel each other (Pearson's  $r=0.8115$ ,  $p<0.05$ ) (see Table 5) whereas Russian teachers' and students' performance significantly parallel each other (Pearson's  $r=0.9526$ ,  $p<0.01$ ) with unexpected “reverse” results on Data and Chance (see Table 6). Also, we found the US teachers' and students' performance on Cognitive Domain significantly parallel each other (Pearson's  $r=0.9993$ ,  $p<0.01$ ) whereas correlation between Russian teachers' and students' performance is adequate but not significant (Pearson's  $r=0.7168$ ).

Moreover, we identified that there is no significant difference between Russian and US teachers' knowledge on Number and Geometry domains (*Chi-square* 0.347  $p>.05$  and *Chi-square* 1.293  $p>.05$ ) (see Table 7). However, there is a statistically significant difference between Russian and US teachers' knowledge on Algebra domain (in favor of Russian teachers; *Chi-square* 6.311  $p<.05$ ) and Data and Chance domain (in favor of US teachers; *Chi-square* 8.003  $p<.05$ ) (see Table 7). This finding closely parallels the US and Russian *students' performance* on TIMSS with a statistically significant difference on Algebra domain (in favor of Russian students) and a difference on Data and Chance domain (in favor of US students).

Also, this study reported that there is no significant difference between Russian and US teachers' knowledge on Knowing and Applying cognitive domains (*Chi-square* 1.707  $p>.05$  and *Chi-square* 0.008  $p>.05$ ) whereas there is a statistically significant difference on Reasoning domain (in favor of Russian teachers; *Chi-square* 19.117  $p<.05$ ) (see Table 8). This finding parallels (not significantly though) the US and Russian students' performance on TIMSS' cognitive domain.

We are cognizant of the limitations concerning the use of secondary data (TIMSS data). Teacher selection was conducted using convenient sampling technique that influences generalizability of the study results. Moreover, there is no cluster matching between teachers participating in the study and students tested in TIMSS.

**Significance.** Despite the limitations, the study main results suggest that student performance on international tests could be explained by teacher knowledge. The study also presents opportunities for comparing, sharing, and learning about issues in cross-national context in US and Russian teacher education, training and development. Moreover, cross-national study on teacher knowledge may inform the field on priorities placed on lower secondary mathematics teachers' knowledge in USA and Russia by content and cognitive domains.

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### Tables

Table 1. US Teachers' means scores by Content Domain

<b>Content Domain</b>	<b>Mean</b>	<b>SE</b>	<b>SD</b>	<b>Conf. level (95%)</b>
Number	623	20.3129	205.1512	40.296
Algebra	563	23.2356	234.6679	46.093
Geometry	514	25.4349	256.8802	50.456
Data and Chance	593	20.9738	211.8252	41.606

Table 2. Russian Teachers' means scores by Content Domain

<b>Content Domain</b>	<b>Mean</b>	<b>Stand. Error</b>	<b>Stand. Deviation</b>	<b>Conf. Level (95%)</b>
Number	656	106.5819	319.7456	23.873
Algebra	728	82.8841	248.6523	30.648
Geometry	586	72.7004	218.1013	45.505
Data and Chance	387	125.0891	306.4044	35.844

Table 3. US Teachers' means scores by Cognitive Domain

<b>Cognitive Domain</b>	<b>Mean</b>	<b>SE</b>	<b>SD</b>	<b>Conf. level (95%)</b>
Knowing	734	19.7673	197.6733	39.2226
Applying	505	20.7101	207.1015	41.0934
Reasoning	495	23.8130	238.1303	47.2502

Table 4. Russian Teachers' means scores by Cognitive Domain

<b>Cognitive Domain</b>	<b>Mean</b>	<b>SE</b>	<b>SD</b>	<b>Conf. level (95%)</b>
Knowing	760	14.2486	135.1745	28.3117
Applying	504	12.7961	121.3950	25.4257
Reasoning	593	17.7406	168.3028	35.2503

Table 5. US Teachers' and Students' Performance by Content Domain

Content Domain	Number	Algebra	Geometry	Data & Chance
Teachers	623	563	514	593
Students	514	512	485	527

Table 6. Russian Teachers' and Students' Performance by Content Domain

Content Domain	Number	Algebra	Geometry	Data & Chance
Teachers	656	728	586	387
Students	534	556	533	511

Table 7. Russian and US Teachers' Knowledge by Content Domain

Content Domain	Number	Algebra	Data and Chance	Geometry
Russia	656	728	387	586
USA	623	563	593	514
Chi-square	0.347	<b>6.311*</b>	<b>8.003**</b>	1.293
p-value	0.5558	0.0119	0.0047	0.2555

Table 8. Russian and US Teachers' Knowledge by Cognitive Domain

Cognitive Domain	Knowing	Applying	Reasoning
Russia	760	504	593
USA	734	505	495
Chi-square	1.707	0.008	<b>19.117**</b>
p-value	0.1914	0.9287	0