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PROFESSIONAL ENGLISH FOR MATHEMATICS
Part I

Учебно-методическое пособие

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Предисловие

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Целью настоящего методического пособия является развитие, углубление и расширение навыков правильного понимания и перевода научной неадаптированной литературы по специальности.

Пособие состоит из двух частей. Первая часть содержит 5 научных текстов; каждый текст сопровождается лексико-грамматическими заданиями. Все задания по своей структуре идентичны и имеют четкие формулировки, что отвечает принципу систематичности расположения материала. Вторая часть представляет собой математический глоссарий, который частично студенты должны составить самостоятельно.

Материал, безусловно, будет полезен и тем, кто самостоятельно хочет освоить навыки работы со специальной литературой.

CONTENT

PART I. MATHEMATICAL TEXTS

TEXT I. MATHEMATICS IS THE LANGUAGE OF SCIENCE

1.1. Fill the first two columns of the table according to the instruction. Then read the text and fill the third column.

Instructions

KNEW – the information that you knew before reading the text
WOULD LIKE TO KNOW- the information that you would like to know
HAVE KNOWN – the information that you have known after reading the text

KNEW	WOULD LIKE TO KNOW	HAVE KNOWN
1		
2		
3		

If people do not believe that mathematics is simple,

It is only because they do not realize how complicated life is.

John Louis von Neumann

One of the foremost reasons given for the study of mathematics is to use a common phrase, that – mathematics is the language of science. This is not meant to imply that mathematics is useful only to those who specialized in science. No, it implies that even a layman must know something about the foundations, the scope and the basic role played by mathematics in our scientific age.

The language of mathematics consists mostly of signs and symbols, and, in a sense, is an unspoken language. There can be no more universal or more simple language, it is the same throughout the civilized world, though the people of each country translate it into their own particular spoken language. For instance, the symbol 5 means the same to a person in England, Spain, Italy or any other country; but in each country it may be called by a different spoken word. Some of the best known symbols of mathematics are the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 and the signs of addition (+), subtraction (-), multiplication (x), division (:), equality (=) and the letters of the alphabets: Greek, Latin, Gothic and Hebrew (rather rarely).

Symbolic language is one of the basic characteristics of modern mathematics for it determines its true aspect. With the aid of symbolism mathematicians can make transition in reasoning almost mechanically by the eye and leave their mind free to grasp the fundamental ideas of the subject matter. Just as music uses symbolism for the representation and communication of sounds, so mathematics expresses quantitatively relations and spatial forms symbolically. Unlike the common language, which is the product of custom, as well as social and political movements, the language of mathematics is carefully, purposefully and often ingeniously designed. By virtue of its compactness, it permits a mathematician to work with ideas which when expressed in terms of common language are unmanageable. This compactness makes for efficiency of thought.

Mathematic language is precise and concise, so that it is often confusing to people unaccustomed to its forms. The symbolism used in math language is essential to distinguish meanings often confused in common speech. Math style aims at brevity and formal perfection. Let us suppose we wish to express in general terms the Pythagorean Theorem, well-familiar to every student through his high-school studies. We may say: "We have a right triangle. If we construct two squares each having an arm of the triangle as a side and if we construct a square having the hypotenuse of the triangle for its side, then the area of the third square is equal to the sum of the areas of the first two". But no mathematician expresses himself that way. He prefers: "The sum of the squares on the sides of a right triangle equals the square on the hypotenuse." In symbols this may be stated as follows: $c^2=a^2+b^2$. This economy of words makes for conciseness of presentation, and math writing is remarkable because it encompasses much in few words. In the study of mathematics much time must be devoted 1) to the expressing of verbally stated facts in math language, that is, in the signs and symbols of mathematics; 2) to the translating of math expressions into common language. We use signs and symbols for convenience. In some cases the symbols are abbreviations of words, but often they have no such relations to the thing they stand for. We cannot say why they stand for what they do; they mean what they do by common agreement or by definition.

The student must always remember that the understanding of any subject in mathematics presupposes clear and definite knowledge of what precedes. This is the reason why "there is no royal road" to mathematics and why the study of mathematics is discouraging to weak minds, those who are not able to master the subject.

1.2. Which of these statements are true? Correct the false ones.

1. Symbolic language is one of the main characteristics of modern mathematics for it determines its true aspect.
2. The language of mathematics consists of signs and symbols.
3. In the process of studying the mathematics much attention should be devoted: 1- to the expressing of verbally stated facts in math language; 2 - to the translating of math expressions into common language.
4. Like the common language, the language of mathematics is carefully, purposefully and often ingeniously designed.
5. Mathematic language is precise and concise, so that it is often confusing to people unaccustomed to its forms.

1.3. Make the false statements negative. Paraphrase, if possible, the negative sentences in more than one way.

Model 1: *Mathematicians define this basic term.*

*Mathematicians **do not define** this basic term.*

***No** mathematician defines this basic term. (Ни один…не…).*

***Don't** mathematicians define this basic term? (Разве … не?).*

1. There can be more universal or more simple language.
2. Symbolic language is the only basic characteristics of modern mathematics.
3. The language of mathematics is randomly designed.

4. The sum of the squares on the sides of a right triangle equals the cube on the hypotenuse.
5. In the study of mathematics much time must be devoted to the learning of multiplication tables.

1.4. Match the following words from the text to form word partnerships, translate them into Russian.

language of		presentation
right		age
conciseness of		mathematics
signs and symbols of		science/mathematics
scientific		triangle

1.5. Find English equivalents of the following phrases:

- понять основные идеи;
- нет смысла отрицать;
- цивилизованный мир;
- выражать количественные отношения;
- гипотенуза треугольника;
- даже обыватель;
- основная причина;
- квадрат гипотенузы;
- социальные и политические преобразования;
- пространственная форма.

1.6. Reorder the words to make a sentence:

1. Of, symbols, the, consists, and, signs, language, of, mathematics, mostly.
2. Characteristics, modern, is, symbolic, its, the, language, aspect, one, determines, of, basic, mathematics, for, of, it, true.
3. The, common, to, often, symbolism, is, used, in, speech, math, language, distinguish, essential, in, meanings, confused.

4. The, what, of, presupposes, understanding, definite, clear, of, subject, mathematics clear, and, any, knowledge, proceeds, in.

5. Language, there, or, can, no, more, more, universal, simple, be, language.

1.7. Do the test:

1. In what profession knowledge of mathematics is required?

a) teacher; b) layman; c) mathematician; d) in all professions.

2. The language of mathematics consists mostly of …?

a) terms and tables; b) signs and letters;

c) signs and symbols; d) symbols and letters.

3. The symbol **5** means … to a person in England, Spain, Italy or any other country.

a) different things; b) "hello"; c) an arm; d) the same.

4. What is the sign of addition?

a) (-); b) (}); c) (+); d) (x).

5. What is the sign of subtraction?

a) (-); b) (=); c) (+); d) (x).

6. What is the sign of multiplication?

a) (#); b) (=); c) (>); d) (x).

7. What is the sign of division?

a) (-); b) (=); c) ("); d) (:).

8. What is the sign of equality?

a) (+); b) (=); c) (&); d) (:).

9. The letters of which alphabets are frequently used in mathematics?

a) Greek, Italian, Russian, and Hebrew; b) Latin, Gothic, and Times New Roman;

c) Latin, Gothic, French, and Hebrew; d) Greek, Latin, Gothic, and Hebrew.

10. The study of mathematics is discouraging to … minds, those who are not able to master the subject.

a) weak; b) dark; c) absent; d) blonde.

1.8. Find derivates in the text for the following words. Explain their meaning in Russian:

Verb	***Noun***	***Adjective***
to symbolize		Symbolic
to differ	Difference	
	Supposition	Suppositional
to present		Representational
to compact		Compact

1.9. Translate from Russian in to English.

В такой отрасли математики, как алгебра, неизвестные величины выражаются буквами. Некоторые буквы называются переменными, так как числа, которые они представляют, меняются от уравнения к уравнению. Другие буквы называются константами, потому что ими представлены числа, имеющие постоянную величину, которая никогда не изменяется.

1.10. Write down the summary of the text using phrases from the Attachment I.

TEXT II. THE HISTORY OF ANCIENT MATHEMATICAL SCHOOLS

1.1 Fill the first two columns of the table according to the instruction. Then read the text and fill the third column.

Instructions

KNEW – the information that you knew before reading the text
WOULD LIKE TO KNOW- the information that you would like to know
HAVE KNOWN – the information that you have known after reading the text

KNEW	WOULD LIKE TO KNOW	HAVE KNOWN
1		
2		
3		

We with pleasure understand math...
She delights us like a Lotus flower.
Aristotle

Great minds of Greece such as Thales, Pythagoras, Euclid, Archimede, Apollonius, Eudoxus, etc. produced an amazing amount of first class mathematics. The fame of these mathematicians spread to all corners of the Mediterranean world and attracted numerous pupils. Masters and pupils gathered in schools which though they had few, buildings and no campus were truly centers of learning. The teaching of these schools dominated the entire life of the Greeks.

Despite the unquestioned influence of Egypt and Babylonia on Greek mathematicians, the mathematics produced by the Greeks differed fundamentally from that which preceded it. It was the Greeks who founded mathematics as a scientific discipline. The Pythagorean School was the most influential in determining both the nature and content Greek mathematics. Its leader Pythagoras founded a community which embraced both mystical and rational doctrines.

The original Pythagorean brotherhood (c. 550—300 B. C.) was a secret aristocratic society whose members preferred to operate from behind the scenes and, from there, to rule social and intellectual affairs with an iron hand. Their noble born initiates were taught entirely by word of mouth. Written documentation was not permitted, since anything written might give away the secrets largely responsible for their power. Among these early Pythagoreans were men who knew more about mathematics then available than most other people of their time. They recognized that vastly superior in design and manageability Babylonian base-ten positional numeration system might make computational skills available to people in all walks of life and rapidly democratize mathematics and diminish their power over the masses. They used their own non positional numeration system (standard Greek alphabet supplemented by special symbols). Although there was no difficulty in determining when the symbols represented a number instead of a word, for computation the people of the lower classes had to consult an exclusive group of experts or to use complicated tables and both of these sources of help were controlled by the brotherhood.

For Pythagoras and his followers the fundamental studies were geometry, arithmetic, music, and astronomy. The basic element of all these studies was number not in its practical computational aspect, but as the very essence of their being; they meant that the nature of numbers should be conceived with the mind only. In spite of the mystical nature of much of the Pythagorean study the members of community contributed during the two hundred or so years following the founding of their organization, a good deal of sound mathematics. Thus, in geometry they developed the properties of parallel lines and used them to prove that “the sum of the angles of any triangle is equal to two right angles”. They contributed in a noteworthy manner to Greek geometrical algebra, and they developed a fairly complete theory of proportional though it was limited to commensurable magnitudes, and used it to deduce properties of similar figures. They were aware of the existence of at least three of the regular polyhedral solids, and they discovered the incommensurability of a side and a diagonal of a square.

Details concerning the discovery of the existence of incommensurable quantities are lacking, but it is apparent that the Pythagoreans found it as difficult to accept incommensurable quantities as to discover them. Two segments are commensurable if there is a segment that "measures" each of them – that is, it contains exactly a whole number of times in each of the segments.

1.2. Which of these statements are true? Correct the false ones.

1. It was the Egyptians who founded mathematics as a scientific discipline.
2. The Pythagorean School was the most influential in determining both the nature and content Greek mathematics.
3. There was some difficulty in determining when the symbols represented a number instead of a word.
4. For Pythagoras the fundamental studies were geometry, arithmetic, music, and astronomy.
5. Two segments are commensurable if there is a segment that "measures" each of them.

1.3. Match the following words from the text to form word partnerships, translate them into Russian.

unquestioned		quantities
written		symbols
Greek		documentation
special		influence
incommensurable		alphabet

1.4. Find English equivalents of the following phrases:

- вычислительные навыки;
- уменьшить чью-то власть;

- система исчисления;
- существование несоизмеримых количеств;
- быть уверенным в существовании;
- схожие фигуры;
- любой треугольник;
- своя собственная;
- управлялась братством;
- свойства параллельных линий.

1.5. Find derivates in the text for the following words and add the absent forms. Explain their meaning in Russian:

Verb	***Noun***	***Adjective/Adverb***
to learn		
		regular
	mystery	
		computational
	Measure	

1.6. Put the article (the, a (an)) where it is necessary.

1. It was … Greeks who founded mathematics as a scientific discipline.
2. … fame of these mathematicians spread to all corners of … Mediterranean world and attracted … numerous pupils.
3. … teaching of these schools dominated … entire life of … Greeks.
4. Among these early … Pythagoreans were … men who knew more about mathematics then available than most other people of their time.

1.7. Do the test:

1. Who produced an amazing amount of first class mathematics?

a) Thales, Pythagoras; b) Newton, Euclid;

c) Archimede, Apollo; d) Apollonius, Eugene Onegin.

2. What civilizations had a great influence on Greek mathematics?

a) Rome and Russia; b) Egypt and Rome;

c) Egypt and Babylonia; d) Babylonia and Rome.

3. The **…** was the most influential in determining both the nature and content Greek mathematics.

a) Pythagoras School; b) Pythagorean School;

c) Pythagorian School; d) Pythagorative School.

4. The original Pythagorean brotherhood (550—300 B. C.) was a secret … society.

a) poor; b) aristocratic; c) mathematical; d) physical.

5. What symbols did the Pythagoreans use in their own system of calculation?

a) points; b) numerals; c) Latin alphabet; d) standard Greek alphabet.

6. Pythagorean brotherhood contributed in a noteworthy manner to Greek ... .

a) geometry; b) algebraic geometry; c) algebra; d) geometrical algebra.

7. What did the Pythagoreans discover about a square?

a) the incommensurability of a side and a diagonal;

b) the incommensurability of area and volume;

c) the incommensurability of a side and a roof;

d) the incommensurability of a diagonal and volume.

8. The Pythagoreans found it as difficult to accept … quantities as to discover them.

a) uncountable; b) commensurable; c) countable; d) incommensurable.

9. What must exist two segments to be comparable?

a) a segment that "describes" each of them;

b) a segment that "covers" each of them;

c) a segment that "denies" each of them;

d) a segment that "measures" each of them.

10. Among the early Pythagoreans were men who knew more about … .

a) mathematics; b) life; c) physics; d) geometry.

1.8. Form the plural: mind, mathematician, this, school, doctrine, member, his, man, person, mass.

1.9. Consult the dictionary and write down the given verbs with their basic meanings into your vocabulary copybook. Collect the family-related words with the same root, i.e. nouns, adjectives, adverbs and set phrases in the dictionary.

Model: to relate (v) - *related (adj) – relative (n) – relatively (adv) – in (with) relation to … (exp).*

To calculate	To miss	To hold
To count	To mention	To measure
To compete	To invent	To direct
To record	To mean	To compute
To provide	To found	To research

1.10. Write down the summary of the text using phrases from the Attachment I.

TEXT III. FIELDS OF MATHEMATICS

1.1 Fill the first two columns of the table according to the instruction. Then read the text and fill the third column.

Instructions

KNEW – the information that you knew before reading the text
WOULD LIKE TO KNOW- the information that you would like to know
HAVE KNOWN – the information that you have known after reading the text

KNEW	WOULD LIKE TO KNOW	HAVE KNOWN
1		
2		
3		

Investment in knowledge always pays the best interest.

Benjamin Franklin

Mathematics can be subdivided into the study of structure, quantity, space, and change. There are also subdivisions dedicated to exploring links from mathematics to other fields: to logic, to set theory (foundations), to the empirical mathematics of the various sciences (applied mathematics), and more recently to the rigorous study of uncertainty.

The study of quantity begins with numbers, first the familiar natural numbers and integers ("whole numbers") and arithmetical operations on them, which are characterized in arithmetic. The deeper properties of integers are studied in number theory, from which come such popular results as Fermat's Last Theorem.

As the number system is further developed, the integers are recognized as a subset of the rational numbers ("fractions"). These, in turn, are contained within the real numbers, which are used to represent continuous quantities. Real numbers are generalized to complex numbers.

Discussion of the natural numbers leads to the transfinite numbers, which formalize the concept of "infinity". Another area of study is size, which leads to the cardinal numbers and then to another conception of infinity: the aleph numbers, which allow meaningful comparison of the size of infinitely large sets.

Many mathematical objects, such as sets of numbers and functions, exhibit internal structure. The structural properties of these objects are investigated in the study of groups, rings, fields and other abstract systems, which are themselves such objects. This is the field of abstract algebra. An important concept here is that of vectors, generalized to vector spaces, and studied in linear algebra. The study of vectors combines three of the fundamental areas of mathematics: quantity, structure, and space. A number of ancient problems concerning Compass and straightedge constructions were finally solved using Galois theory.

The study of space originates with geometry – in particular, Euclidean geometry. Trigonometry is the branch of mathematics that deals with relationships between the sides and the angles of triangles and with the trigonometric functions; it combines space and numbers, and encompasses the well-known Pythagorean theorem. The modern study of space summarizes these ideas to include higher-dimensional geometry, non-Euclidean geometries and topology. Quantity and space both play a role in analytic geometry, differential geometry, and algebraic geometry. Within differential geometry are the concepts of fiber bundles and calculus on manifolds, in particular, vector and tensor calculus. Within algebraic geometry is the description of geometric objects as solution sets of polynomial equations, combining the concepts of quantity and space, and also the study of topological groups, which combine structure and space. Lie groups are used to study space, structure, and change. Topology in all its many ramifications may have been the greatest growth area in 20th century mathematics; it includes point-set topology, set-theoretic topology, algebraic topology and differential topology. In particular, instances of modern day topology are metrizability theory, axiomatic set theory, homotopy theory, and Morse theory. Topology also includes the now solved Poincaré conjecture and the controversial four color theorem, whose only proof, by computer, has never been verified by a human.

To understand and describe change is a common theme in the natural sciences, and calculus was developed as a powerful tool to investigate it. Functions arise here, as a central concept describing a changing quantity. The rigorous study of real numbers and functions of a real variable is known as real analysis, with complex analysis the equivalent field for the complex numbers.

Functional analysis focuses attention on (typically infinite-dimensional) spaces of functions. One of many applications of functional analysis is quantum mechanics. Many problems lead naturally to relationships between a quantity and its rate of change, and these are studied as differential equations. Many phenomena in nature can be described by dynamical systems; chaos theory makes precise the ways in which many of these systems exhibit unpredictable yet still deterministic behavior.

1.2. Make a plan according to the text.

1.3. Which of these statements are true? Correct the false ones.

1. Mathematics can be subdivided into the study of quantity, structure, space, and change.
2. Understanding and describing number is a common theme in the natural sciences, and calculus was developed as a powerful tool to investigate it.
3. Trigonometry is the branch of mathematics that deals with relationships between the shapes and the angles of triangles and with the trigonometric functions; it combines space and numbers, and encompasses the well-known Pythagorean theorem.
4. The study of quantity starts with numbers, first the familiar natural numbers and integers and arithmetical operations on them, which are characterized in arithmetic.
5. Many mathematical objects, such as sets of numbers and functions, exhibit internal form.

1.4. Match the following words from the text to form word partnerships, translate them into Russian.

internal		change
unpredictable		comparison
fiber		structure
meaningful		bundle
rate of		behavior

1.5. Find English equivalent of the following phrases:

- целое число;
- точное доказательство;
- бесконечное множество;
- множество функций;
- включать в себя;
- стороны и углы треугольника;
- музыка для души;
- пространство функций.

1.6. Find word combinations with the following words from the text.

1. empirical
2. applied
3. polynomial
4. continuous
5. structural
6. natural
7. equivalent
8. quantum
9. differential
10. general

1.7. Choose one sentence from the Text and make up 5 types of questions to it.

Model: Many trends and traditions in this search are mixed.

Q1 (General): *Are many trends and traditions in this search mixed?*

Q2 (Alternative): *Are many trends and traditions in this search mixed or combined?*

Q3 (Tag): *Many trends and traditions in this search are mixed, aren't they?*

Q4 (Special): *Where are many trends and traditions mixed?*

Q5 (To the subject): *What in this search is mixed?*

1.8. Do the test:

1. The deeper properties of integers are studied in number theory, from which come such popular results as Fermat's … Theorem.

a) Third; b) Last; c) First; d) Second.

2. Many mathematical objects, such as sets of … , exhibit internal structure.

a) figures and numbers; b) shapes and functions;

c) numbers and functions; d) functions and space.

3. … both play a role in analytic geometry, differential geometry, and algebraic geometry.

a) space and shape ; b) quantity and figures; c) quantity and space; d) quantity and numbers.

4. Real numbers are generalized to … numbers.

a) complex; b) simple; c) "whole" ; d) - .

5. Functional analysis focuses attention on … of functions.

a) type; b) mode; c) structure; d) spaces.

6. The modern study of space generalizes these ideas to include higher-dimensional geometry, non-Euclidean geometries and … .

a) arithmetic; b) Euclidian geometries; c) topology; d) analysis.

7. Topology also includes the now solved Poincaré conjecture and the controversial four color theorem, whose only proof, by computer, has never been … .

a) checked by a computer; b) proven by a human;

c) verified by a human; d) analyzed by a human .

8. Within … geometry is the description of geometric objects as solution sets of polynomial equations.

a) algebraic; b) Euclidean; c) analytic ; d) differential .

9. Many phenomena in nature can be described by … systems.

a) similar; b) dynamical; c) static; d) different.

10. Another area of study is … , which leads to the cardinal numbers and then to another conception of infinity.

a) size; b) shape; c) form; d) space.

1.9. Translate from Russian into English.

Математика – это наука о числах и количествах, о структурах, порядках и отношениях, что в нее входят арифметика и алгебра, геометрия и тригонометрия, и т.д. Математика в отличие от естественных наук, изучает не явления природы, а логические построения, поэтому эксперименты в математике являются не испытанием природы, а испытанием гипотез в условиях логики. Счет стал началом математики. Люди с математикой сталкивались с давних времен. Например, чтобы определить, кто богаче? Или у кого больше скота? Родоначальниками математики признаны греки (6–4 вв. до н.э.). Математика делилась на арифметику и логистику. В Средние века (около 400–1100) уровень математического знания не поднимался выше арифметики, но важным разделом математики в тот период считалась астрология. В Западной Европе в 16 веке были введены в обращение десятичные дроби и правила арифметических действий с ними. В начале 19 века математиков продолжала занимать основная задача алгебры – поиск общего решения алгебраических уравнений. Ни один математик сегодня не может надеяться знать больше того, что происходит в очень маленьком уголке науки.

1.10. Write down the summary of the text using phrases from the Attachment I.

TEXT IV. MATHEMATICAL PROBLEMS

1.1. Fill the first two columns of the table according to the instruction. Then read the text and fill the third column.

Instructions

KNEW – the information that you knew before reading the text
WOULD LIKE TO KNOW- the information that you would like to know
HAVE KNOWN – the information that you have known after reading the text

KNEW	WOULD LIKE TO KNOW	HAVE KNOWN
1		
2		
3		

Every solution breeds new problems.

Murphy's law

Part I. Mathematical problems

(an extract from the lecture of D. Hilbert, Paris 1900)

History teaches the continuity of the development of science. We know that every age has its own problems, which the following age either solves or casts aside as profitless and replaces by new ones. If we would obtain an idea of the probable development of mathematical knowledge in the immediate future, we must let the unsettled questions pass before our minds and look over the problems which the science of today sets and whose solution we expect from the future. To such a review of problems the present day, lying at the meeting of the centuries seems to be well adapted. For the close of a great epoch not only invites us to look back into the past but also directs our thoughts to the unknown future.

The deep significance of certain problems for the advance of mathematical science in general and the important role which they play in the work of the individual investigator are not to be denied. As long as a branch of science offers an abundance of problems, so long is it alive; a lack of problems foreshadows extinction or the cessation of independent development. Just as every human undertaking pursues certain objects, so also mathematical research requires its problems. It is by the solution of problems that the investigator tests the temper of his steel; he finds new methods and new outlooks, and gains a wider and freer horizon.

It is difficult and often impossible to judge the value of a problem correctly in advance; for the final award depends upon the gain which science obtains from the problem. Are whether there general criteria which mark a good mathematical problem? An old French mathematician said: "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street." This clearness and ease of comprehension, here insisted on for a mathematical theory, I should still more demand for a mathematical problem if it is to be perfect; for what is clear and easily comprehended attracts, the complicated repels us.

Moreover a mathematical problem should be difficult in order to entice us, yet not completely inaccessible, lest it mock at our efforts. It should be to us a guide post on the mazy paths to hidden truths, and ultimately a reminder of our pleasure in the successful solution.

The mathematicians of past centuries were accustomed to devote themselves to the solution of difficult particular problems with passionate zeal. They knew the value of difficult problems. For example, the "problem of the line of quickest descent," proposed by John Bernoulli. Experience teaches, explains Bernoulli, that lofty minds are led to strive for the advance of science by nothing more than by laying before them difficult and at the same time useful problems, and he therefore hopes to earn the thanks of the mathematical world by following the example of men like Mersenne, Pascal, Fermat, Viviani and others and laying before the distinguished analysts of his time a problem by which, as a touchstone, they may test the value of their methods and measure their strength. The calculus of variations owes its origin to this problem of Bernoulli and to similar problems.

1.2. Put the statements in the right order.

1. Mathematics helps to find solutions to various problems.
2. The solution of the problem depends on the perspective of privilege that will get the science.
3. There are different problems in each time of our life.
4. Mathematical problem should be difficult in order to entice us.
5. Lack of problems foreshadows extinction or the cessation of independent development.

1.3. Match the words opposite in meaning.

1. difficult	a. mess
2. clearness	b. starting
3. problem	c. independent
4. lack	d. well-known
5. dependent	e. easy
6. final	f. prevent
7. order	g. secondary
8. help	h. plenty
9. general	i. misunderstanding
10. unknown	j. solution

Part II. The solution of mathematical problems

(Friedrichs, K.O. "Mathematical aspects of flow problems of hyperbolic type)

To solve a mathematical problem meant to find its complete numerical solution. Gradually it became clear that such explicit solutions are possible only in exceptional cases, that in general one must be satisfied with a scheme by which the solution may be determined approximately, though with any desired accuracy. Something quite different is very frequently offered as the solution of a mathematical problem, namely, a representation of the solution in terms of the data of the problem; although it is in principle possible to devise a scheme for numerical calculation from such a representation, the question remains: What actually is the solution? Mathematicians, in their search for representations of solutions, often modified the meaning of "solution" even further: to solve a problem is simply to prove the unique existence of solution.

A mathematical problem which possesses a unique solution is referred to as correctly posed or formulated. The way in which a large class of mathematical problems posed is never questioned. These problems are mostly of a standard, rather regular, type. Doubts arise, however, when the actual physical problem is replaced by an idealized problem. Such idealized problems may be considered as limiting cases of actual problems, arising when, for example, the domain is extended to infinity, forces are concentrated on surfaces, lines or points, or terms in the equations are simply omitted as insignificantly small. To the understanding of such idealized problems, purely mathematical existence and uniqueness considerations may still make valuable contributions.

As it is often emphasized, not only existence and uniqueness, but also a third abstract property of the solution should be required of the problem if it is to be called correctly posed: the property of continuous dependence on the data. Since physical data are not given with absolute precision, the mathematical problem is certainly not the appropriate expression of an actual physical situation if an arbitrarily small variation of the data may have a finite effect on the solution, or may destroy its existence or uniqueness. If the solution does not depend continuously on the data, it may be called unstable. There are important problems, which possess solutions only for exceptional values of the data; thus the solutions do not depend continuously on the data even when they exist.

1.4. Which of these statements are true? Correct the false ones.

1. The mathematicians of past centuries were not accustomed to devote themselves to the solution of difficult particular problems.
2. The deep significance of certain problems for the advance of mathematical science in general and the important role which they play in the work of the individual investigator are denied.
3. It is difficult and often impossible to judge the value of a problem correctly in advance.
4. A mathematical problem should be simple in order to entice us.
5. To solve a mathematical problem meant to find its complete numerical solution.
6. Existence and uniqueness of the problem should be required if it is to be called correctly posed.
7. The mathematical problem is the appropriate expression of an actual physical situation.
8. If the solution depends continuously on the data, it may be called unstable.

1.5. Match the following words from the text to form word partnerships, translate them into Russian.

unsettled		science
individual		calculation
branch		existence
abundance of		property
hidden		values
numerical		investigator
unique		problems
a third abstract		questions

exceptional truths

1.6. Find English equivalent of the following phrases:

- развитие науки;
- получить представление;
- обилие проблем;
- предвещать вымирание;
- прекращение самостоятельного развития;
- судить о величине;
- легкость понимания;
- линия наискорейшего спуска;
- стремиться вперед;
- свойство непрерывной зависимости.

1.7. Match the following terms with definitions and translate them into Russian.

a. algebra	**1.** a physical quantity having magnitude and direction, represented by a directed arrow indicating its orientation in space
b. triangle	***2.*** a step by step procedure by which an operation can be carried out
c. arithmetic	***3.*** a proposition that is not actually proved or demonstrated, but is considered to be self-evident and universally accepted as a starting point for deducing and inferring other truths and theorems, without any need of proof
d. coordinate	***4.*** the ordered pair that gives the location or position of a point on a coordinate plane, determined by the point's distance from the *x* and *y* axes
e. axiom	***5.*** the part of mathematics that studies quantity, especially as the result of combining numbers (as opposed to variables) using the traditional operations

	of addition, subtraction, multiplication and division
f. algorithm	***6.*** a polygon with three edges and three vertices, e.g. a triangle with vertices A, B, and C is denoted ΔABC
g. vector	***7.*** a branch of mathematics that uses symbols or letters to represent variables, values or numbers, which can then be used to express operations and relationships and to solve equations

1.8. Do the test:

1. It is by the solution of problems that the investigator … the temper of his steel.

a) tests; b) analyzes; c) evaluates; d)reviews .

2. It is difficult and often impossible to judge … correctly in advance.

a) the relevance of the problem; b) the significance of the problem;

c) the value of problem ; d) the importance of problem.

3. The deep … of certain problems for the advance of mathematical science in general and the … role which they play in the work of the individual investigator are not to be denied.

a) importance, essential; b) meaning, quite important;

c) value, significant; d) significance, important .

4. Who said the following "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street."

a) ancient mathematician; b) modern mathematician;

c) old French mathematician; d) John Bernoulli .

5. Mathematical problem should be a … post on the mazy paths to hidden … .

a) conductor, validity; b)guide, truths ; c)leader, accuracy ; d)direct, true .

6. To solve a mathematical problem meant to find its … … solution.

a) full numerical; b) entire numeral; c)total numerical ; d)complete numerical.

7. What actually is the solution? To solve the problem is … .

a) to deny the existence of solution; b) to prove the unique existence of solution; c)to prove the existence of equation ; d)to confirm the existence of solution .

8. Idealized problems may be considered as … cases of actual problems.

a) limit; b) restrictive; c) limiting; d)finite .

9. The solution do not depend continuously on the … even when they exist.

a) information; b) data; c) evidence; d) facts.

10. If the solution does not depend continuously on the data, it may be called … .

a) unstable; b) unbalanced; c) irregular; d) unsteady.

1.9. Translate from Russian in to English.

Философы и математики (часть 1)

Одновременно с развитием математики появились методологические и философские труды о науке. Примером может служить классификация Геминуса, греческого астронома и математика 1 в. до н.э. Он считал, что наука уже накопила достаточно разнообразных сведений во многих областях.

Согласно изучению Аристотеля, математика изучает свойства, которые можно «абстрагировать» от объектов физического мира. Кроме того, как и все науки, основывающиеся на доказательствах, она строится на определенных принципах, так что одна наука предполагает существование другой, одна подчиняется другой, как говорил Аристотель. Так, например, оптика «подчиняется» геометрии. Что говорит о существовании логически упорядоченной иерархии наук. Такую иерархию следует отличать от принятого у греческих ученых противопоставления "практической" и "чистой" математики. По Аристотелю, только последняя заслуживает того, чтобы ее включили в свободное образование. «Быть свободным» здесь самоцель.

Искусство приукрашивать одерживает верх над прагматизмом технических расчетов: наука для науки становится высшей формой деятельности. По Платону, математика варваров – какого бы высокого уровня развития не достигла их цивилизация – была всего лишь искусством, не освобожденным от пут необходимости. Греческая философия соединила, таким образом, понятия, принадлежащие к различным сферам – методической и философской.

В трактатах по оптике и астрономии применялись принципы геометрии, поскольку с помощью дедуктивного метода можно было легко обойти все, что представлялось «наглядным» и «практическим». Правда, остается неясным, как сами математики относились к такому определению своего рода занятий. Кроме того, не следует переносить современное понятие «чистой» и «прикладной» математики на «невещественную» и «наглядную» математику древних, так как они не совпадают.

Говоря об идеале «бескорыстной» науки, нельзя не затронуть проблему мотивации развития математики. Здесь нужно различать явления, игравшие роль внешних факторов, от тех, которые можно назвать внутренними. В первой группе следует выделить оптику и астрономию, которые мы относим к физике. А ученые древности относили к области математики. Сюда же относится статика, учение о равновесии.

1.10. Write down the summary of the text using phrases from the Attachment I.

TEXT V. COMPASS AND CONSTRUCTIONS

1.1 Fill the first two columns of the table according to the instruction. Then read the text and fill the third column.

Instructions

KNEW – the information that you knew before reading the text
WOULD LIKE TO KNOW- the information that you would like to know
HAVE KNOWN – the information that you have known after reading the text

KNEW	WOULD LIKE TO KNOW	HAVE KNOWN
1		
2		
3		

Avoiding the phrase "I don't have time...",
will soon help you to realize that you do have the time needed
for just about anything you choose to accomplish in life.
Bo Bennett

A "construction" is drawing geometric figures with a high degree of accuracy. The construction performed constitutes both a proof of the existence of a geometric object and the solution of the problem. The ancient Greeks were convinced that all plane figures can be constructed with a compass and a straightedge alone. Their methods of bisecting a line and an angle are ingenious and hard to improve on. They worked with all numbers geometrically. A length was chosen to represent the number 1, and all other numbers were expressed in terms of this length. They solved equations with unknowns by series of geometric constructions. The answers were line segments whose length were the unknown value sought. The Greeks imposed the restrictions of straightedge and compass for the construction of the problems. It is supposed that this tradition was started by Plato Greece’s greatest philosopher. He claimed that more complicated instruments called for manual skill unworthy of a thinker. The Greeks failed to obtain the solution of the famous problems under the restrictions specified not due to the lack of ingenuity of the geometers. The Greeks’ persistent efforts to find compass-and-straightedge ways of trisecting an angle, squaring the circle and duplicating the cube were not futile for almost 2000 years. The Greeks made great math discoveries on the way. The desire to gain full understanding of the theoretical character of the problems inspired many great mathematicians- among them Descartes, Gauss, Poncelet, Lindemann – to mention but a few. The long years of labor on these “impractical”, “worthless” problems indicate the care, patience, persistence and rigor of mathematicians in their attempts to perform the constructions and justify them theoretically. The problems did not exhaust themselves. Even nowadays some authors of the scientific papers issued “solutions” containing some fallacies. The search for the rigorous solution resulted in great discoveries and novel developments in mathematics.

1.2. Which of these statements are true? Correct the false ones.

1. The ancient Egyptians were convinced that all plane figures can be constructed with a compass and a straightedge alone.
2. The Greeks succeeded to obtain the solution of the famous problems under the restrictions specified not due to the lack of ingenuity of the geometers.
3. The problems did not exhaust themselves.
4. The construction constitutes a proof of the existence of a geometric object.
5. The Greeks imposed the restrictions of straightedge and compass for the construction of the problems.

1.3. Match the following words from the text to form word partnerships, translate them into Russian.

to bisect		value
unknown		papers

equations with		solution
scientific		a line and an angle
rigorous		unknowns

1.4. Find English equivalents of the following phrases:

- высокая степень точности;
- сложно доказать;
- отсутствие изобретательности;
- строгость математики;
- плоские фигуры;
- квадратура круга;
- попытки отыскать;
- получить полное представление;
- доказательство существования;
- обосновать теоретически.

1.5. Do the test:

1. A "…" is drawing geometric figures with a high degree of accuracy.

a) creation; b) arrangement; c) construction ; d)structure .

2. The ancient … were convinced that all plane figures can be constructed with a compass and a straightedge alone.

a) Greeks; b) times; c) Pythagoreans; d) Babylonians.

3. Greeks' methods of bisecting a line and an angle are … and hard to improve on.

a) important; b) great; c) original; d) ingenious.

4. The Greeks … to obtain the solution of the famous problems under the restrictions specified not due to the lack of ingenuity of the geometers.

a) succeeded; b) failed; c) were not able; d) cannot.

5. The Greeks' efforts to find compass-and-straightedge ways of trisecting an angle, squaring the circle and duplicating the cube were not futile for almost … .

a) 200 years; b) 100 years; c) 1000 years; d) 2000 years.

6. A length was chosen to represent the number … , and all other numbers were expressed in terms of this length.
a) 2; b) 3; c) 1; d) 4.

7. The desire to gain full understanding of the … … of the problems inspired many great mathematicians.
a) practical character; b) mathematical character;
c) theoretical and practical character; d) theoretical character.

8. The search for the … … resulted in great discoveries and novel developments in mathematics.
a) accurate solution; b) rigorous solution ;
c) strict decision; d) rigorous decision.

9. Nowadays some authors of the scientific papers issued “solutions” containing some … .
a) fallacies; b) mistakes; c) false conclusions; d) reliable results.

10. Who of these great mathematicians are mentioned in the text?
a) Descartes, Poncelet, Lindemann; b) Descartes, Gauss, Poncelet;
c) Lindemann, Descartes, Gauss; d) Gauss, Lindemann, Descartes, Poncelet.

1.6. Translate from Russian in to English.

Философы и математики (часть 2)

Что нам известно о "внутренней" мотивации? Можно попробовать найти ее определение в предисловиях, которыми математики, начиная с Архимеда, предваряли свои сочинения. Оказывается, что «бескорыстные» исследования вовсе не плод греческого стереотипа мышления. Они предполагают существование некоего сообщества математиков, которые следуют установленным нормам.

Прежде всего, эти ученые считают нужным оправдываться в том, что они занимаются наукой ради науки, им это кажется вполне естественным. В лучшем случае они лишь уточняют, почему выбрали именно математику, а не физику или теологию. Математика более достоверна и строга, ее предмет более «постоянен», чем физика, и более «доступен», чем теология.

В Древней Греции математики составляли своего рода «международное» сообщество, члены которого были рассеяны по всему Средиземноморью: в Греции, Малой Азии, Египте и на Сицилии. Они поддерживали личные контакты и обменивались своими работами. Прежде всего, ученые стремились передать коллегам свои задачи, найти решения тех задач, которые присылали им, или подвергнуть критике неудачные решения, предложенные другими. Так, некоторые из них приобретали общепризнанный авторитет: им присылали на отзыв научные труды, они, в свою очередь, рассылали их самым, по их мнению, достойным. Попадались среди них и самозванцы, но разоблачить обман было легко: им предлагали задачу, не имеющую решения, а они уверяли, что решили ее. Конечно, такие контакты оставались сугубо личными, они совсем не похожи на отношения, которые складываются между учеными в рамках современных институтов.

«Бескорыстная» наука, таким образом, связывалась с существованием некоей группы, внутри которой царило соперничество, напоминающее то, что происходит среди современных ученых. Впрочем, такое сравнение не вполне правомерно, слишком уже ощутима разница масштабов этих сообществ: в эпоху эллинизма число ученых, особенно математиков, не превышало нескольких сотен. Во время римского владычества лучшие авторы (Птолемей, Папп) занимались уже только уточнением полученных результатов. Соперничество и поиск нового ушли в прошлое вместе с породившей их эпохой.

1.7. Write down the summary of the text using phrases from the Attachment I.

PART II. GLOSSARY OF SOME MATHEMATICAL TERMS

Task: Read the definitions and guess what the term is in each section. Write down the terms and make your own Glossary.

A

________: a branch of mathematics that uses symbols or letters to represent variables, values or numbers, which can then be used to express operations and relationships and to solve equations

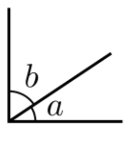
________ ________: a combination of numbers and letters equivalent to a phrase in language, e.g. $x^2 + 3x - 4$

________ ________: a combination of numbers and letters equivalent to a sentence in language, e.g. $y = x^2 + 3x - 4$

________: a step by step procedure by which an operation can be carried out

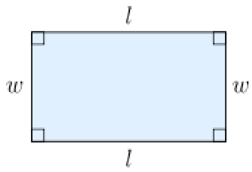
________ ________: pairs of numbers for which the sum of the divisors of one number equals the other number, e.g. 220 and 284, 1184 and 1210

________ ________: grounded in the rigorous formulation of calculus, analysis is the branch of pure mathematics concerned with the notion of a limit (whether of a sequence or of a function)

________: the figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle, e.g. 

________: the part of mathematics that studies quantity, especially as the result of combining numbers (as opposed to variables) using the traditional operations of addition, subtraction, multiplication and division (the more advanced manipulation of numbers is usually known as number theory)

________: is anything that is similar but not exactly equal to something else, e.g. $\pi \approx 3{,}14$

________: the quantity that expresses the extent of a two-dimensional figure or shape, or planar lamina, in the plane, e.g. 

(the area of rectangle)

________ ________: property (which applies both to multiplication and addition) by which numbers can be added or multiplied in any order and still yield the same value, e.g. $(a + b) + c = a + (b + c)$ or $(ab)c = a(bc)$

________: a line that the curve of a function tends towards as the independent variable of the curve approaches some limit (usually infinity) i.e. the distance between the curve and the line approaches zero

________: a proposition that is not actually proved or demonstrated, but is considered to be self-evident and universally accepted as a starting point for deducing and inferring other truths and theorems, without any need of proof

B

________ ***n*****:** the number of unique digits (including zero) that a positional numeral system uses to represent numbers, e.g. base 10 (decimal) uses 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 in each place value position; base 2 (binary) uses just 0 and 1; base 60 (sexagesimal, as used in ancient Mesopotamia) uses all the numbers from 0 to 59; etc

________ ________: a popular interpretation of probability which evaluates the probability of a hypothesis by specifying some prior probability, and then updating in the light of new relevant data

________: a one-to-one comparison or correspondence of the members of two sets, so that there are no unmapped elements in either set, which are therefore of the same size and cardinality

________: a polynomial algebraic expression or equation with just two terms, e.g. $2x^3 - 3y = 7$; $x^2 + 4x$; etc

C

________ (________ ________): a branch of mathematics involving derivatives and integrals, used to study motion and changing values

________ ________: an extension of calculus used to search for a function which minimizes a certain functional (a functional is a function of a function)

________ ________: numbers used to measure the cardinality or size (but not the order) of sets - the cardinality of a finite set is just a natural number indicating the number of elements in the set; the sizes of infinite sets are described by transfinite cardinal numbers, $\aleph_0$ (aleph-null), $\aleph_1$ (aleph-one), etc.

________ ________: a pair of numerical coordinates which specify the position of a point on a plane based on its distance from the two fixed perpendicular axes (which, with their positive and negative values, split the plane up into four quadrants)

________: the factors of the terms (i.e. the numbers in front of the letters) in a mathematical expression or equation, e.g. in the expression $4x + 5y^2 + 3z$, the coefficients for x, y^2 and z are 4, 5 and 3 respectively

________: the study of different combinations and groupings of numbers, often used in probability and statistics, as well as in scheduling problems and Sudoku puzzles

________ ________: the study of mathematical models and dynamical systems defined by iteration of functions on complex number spaces

________ ________**:** a number expressed as an ordered pair comprising a real number and an imaginary number, written in the form $a + bi$, where a and b are real numbers, and i is the imaginary unit (equal to the square root of -1)

________ ________**:** a number with at least one other factor besides itself and one, i.e. not a prime number

________ ________**:** the section or curve formed by the intersection of a plane and a cone (or conical surface), depending on the angle of the plane it could be an ellipse, a hyperbola or a parabola

________ ________**:** a fraction whose denominator contains a fraction, whose denominator in turn contains a fraction, etc.

________**:** the ordered pair that gives the location or position of a point on a coordinate plane, determined by the point’s distance from the x and y axes, e.g. (2, 3.7) or (-5, 4)

________ ________**:** a plane with two scaled perpendicular lines that intersect at the origin, usually designated x(horizontal axis) and y (vertical axis)

________**:** a measure of relationship between two variables or sets of data, a positive correlation coefficient indicating that one variable tends to increase or decrease as the other does, and a negative correlation coefficient indicating that one variable tends to increase as the other decreases and vice versa

________ ________**:** a polynomial having a degree of 3 (i.e. the highest power is 3), of the form $ax^3 + bx^2 + cx + d= 0$, which can be solved by factorization or formula to find its three roots

________**:** the action of finding the number of elements of a finite set of objects

________**:** a deliberate process that transforms one or more inputs into one or more results, with variable change

________**:** a term with several related but distinct meanings, e.g. in general mathematics, a *correspondence* is an ordered triple (X,Y,R), where R is a relation from X to Y

D

________ ________**:** a real number which expresses fractions on the base 10 standard numbering system using place value, e.g. $^{37}/_{100} = 0.37$

________ ________**:** a type of reasoning where the truth of a conclusion necessarily follows from, or is a logical consequence of, the truth of the premises (as opposed to inductive reasoning)

________**:** a measure of how a function or curve changes as its input changes, i.e. the best linear approximation of the function at a particular input value, as represented by the slope of the tangent line to the graph of the function at that point, found by the operation of differentiation

________ ________**:** an equation that expresses a relationship between a function and its derivative, the solution of which is not a single value but a function (has many applications in engineering, physics economics, etc.)

________**:** the operation in calculus (inverse to the operation of integration) of finding the derivative of a function or equation

________ ________**:** property whereby summing two numbers and then multiplying by another number yields the same value as multiplying both values by the other value and then adding them together, e.g. $a(b + c) = ab + ac$

E

________**:** a member of, or an object in, a set

________: a plane curve resulting from the intersection of a cone by a plane that looks like a slightly flattened circle (a circle is a special case of an ellipse)

________ (_____) ______: a set that has no members, and therefore has zero size, usually represented by {} or ø

________: the relationship between expressions that represent the same value or mathematical object, e.g. the equality between A and B is written $A = B$, and pronounced A equals B

________: a statement of an equality containing one or more variables, e.g. $Ax^2 + Bx + C = y$

________ ________: the amount predicted to be gained, using the calculation for average expected payoff, which can be calculated as the integral of a random variable with respect to its probability measure (the expected value may not actually be the most probable value and may not even exist, e.g. 2.5 children)

________: the mathematical operation where a number (the base) is multiplied by itself a specified number of times (the exponent), usually written as a superscript a^n, where a is the base and n is the exponent, e.g. $4^3 = 4 \times 4 \times 4$

F

________: a number that will divide into another number exactly, e.g. the factors of 10 are 1, 2 and 5

________: the product of all the consecutive integers up to a given number (used to give the number of permutations of a set of objects), denoted by $n!$, e.g. $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$

________ ________: prime numbers that are one more than a power of 2 (and where the exponent is itself a power of 2), e.g. 3 ($2^1 + 1$), 5 ($2^2 + 1$), 17 ($2^4 + 1$), 257 ($2^8 + 1$), 65,537 ($2^{16} + 1$), etc

________ ________: a method of approximating the derivative or slope of a function using approximately equivalent difference quotients (the function difference divided by the point difference) for small differences

________ ________: a set that has a finite number of elements, e.g. {2, 4, 6, 8, 10}

________: a rule or equation describing the relationship of two or more variables or quantities, e.g. $A = \pi r^2$

________: a way of writing rational numbers (numbers that are not whole numbers), also used to represent ratios or division, in the form of a numerator over a denominator, e.g. $^3/_5$ (a unit fraction is a fraction whose numerator is 1)

________: a self-similar geometric shape (one that appears similar at all levels of magnification) produced by an equation that undergoes repeated iterative steps or recursion

________: a relation or correspondence between two sets in which one element of the second (codomain or range) set $f(x)$ is assigned to each element of the first (domain) set x, e.g. $f(x) = x^2$ or $y = x^2$ assigns a value to $f(x)$ or y based on the square of each value of x

G

________ ________: a branch of mathematics that attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others, with applications in the areas of economics, politics, biology, engineering, etc

________ ________: (________ ________, ________ ________): the ratio of two quantities (equivalent to approximately 1 : 1.6180339887) where the ratio of the sum of the quantities to the larger quantity equals the ratio of the larger quantity to the smaller one, usually denoted by the Greek letter phi φ (phi)

________ ________: the mathematical field that studies the algebraic structures and properties of groups and the mappings between them

H

________: a smooth symmetrical curve with two branches produced by the section of a conical surface

I

________: an equality that remains true regardless of the values of any variables that appear within it, e.g. for multiplication, the identity is one; for addition, the identity is zero

________ ________: numbers in the form *bi*, where *b* is a real number and *i* is the "imaginary unit", equal to √-1 (i.e. $i^2 = -1$)

________ ________ or ________: a type of reasoning that involves moving from a set of specific facts to a general conclusion, indicating some degree of support for the conclusion without actually ensuring its truth

________ ________: the sum of an infinite sequence of numbers (which are usually produced according to a certain rule, formula or algorithm)

________: quantities or objects so small that there is no way to see them or to measure them, so that for all practical purposes they approach zero as a limit (an idea used in the development of infinitesimal calculus)

________: a quantity or set of numbers without bound, limit or end, whether countably infinite like the set of integers, or uncountable infinite like the set of real numbers (represented by the symbol ∞)

________: whole numbers, both positive (natural numbers) and negative, including zero

________: the area bounded by a graph or curve of a function and the x axis, between two given values of x (definite integral), found by the operation of integration

________: the operation in calculus (inverse to the operation of differentiation) of finding the integral of a function or equation

________ ________: numbers that cannot be represented as decimals (because they would contain an infinite number of non-repeating digits) or as fractions of one integer over another, e.g. π, $\sqrt{2}$, e

L

________ ________: a method of regression analysis used in probability theory and statistics to fit a curve-of-best-fit to observed data by minimizing the sum of the squares of the differences between the observed values and the values provided by the model

________: the point towards which a series or function converges, e.g. as x becomes closer and closer to zero, ${}^{(\sin x)}/_{x}$ becomes closer and closer to the limit of 1

________: in geometry, a one-dimensional figure following a continuous straight path joining two or more points, whether infinite in both directions or just a line segment bounded by two distinct end points

________ ________: an algebraic equation in which each term is either a constant or the product of a constant and the first power of a single variable, and whose graph is therefore a straight line, e.g. $y = 4$, $y = 5x + 3$

________ ________: a technique in statistics and probability theory for modelling scattered data by assuming an approximate linear relationship between the dependent and independent variables

________: the inverse operation to exponentiation, the exponent of a power to which a base (usually 10 or *e* for natural logarithms) must be raised to produce a given number, e.g. because $1{,}000 = 10^3$, the $\log_{10} 100 = 3$

________: the study of the formal laws of reasoning (mathematical logic the application of the techniques of formal logic to mathematics and mathematical reasoning, and vice versa)

M

________ ________: a square array of numbers where each row, column and diagonal added up to the same total, known as the magic sum or constant (a semi-magic square is a square numbers where just the rows and columns, but not both diagonals, sum to a constant)

________: a rectangular array of numbers, which can be added, subtracted and multiplied, and used to represent linear transformations and vectors, solve equations, etc

________ ________: numbers that are one less than 2 to the power of a prime number, e.g. 3 (2^2 - 1); 7 (2^3 - 1); 31 (2^5 - 1); 127 (2^7 - 1); 8,191 (2^{13} - 1); etc

________ ________: prime numbers that are one less than a power of 2, e.g. 3 (2^2 - 1); 7 (2^3 - 1); 31 (2^5 - 1); 127 (2^7 - 1); 8,191 (2^{13} - 1); etc - many, but not all, Mersenne numbers are primes, e.g. 2,047 = 2^{11} - 1 = 23 x 89, so 2,047 is a Mersenne number but not a Mersenne prime

________ of ________: a method of finding the area of a shape by inscribing inside it a sequence of polygons whose areas converge to the area of the containing shape (a precursor to the methods of calculus)

________: a number by which two given numbers can be divided by integer division, and produce the same remainder, e.g. 38 ÷ 12 = 3 remainder 2, and 26 ÷ 12 = 2 remainder 2, therefore 38 and 26 are congruent modulo 12, or (38 ≡ 26) mod 12

________: an algebraic expression consisting of a single term (although that term could be an exponent), e.g. $y = 7x$, $y = 2x^3$

N

________ ________: the set of positive integers (regular whole counting numbers), sometimes including zero

________ ________: any integer, ration or real number which is less than 0, e.g. -743, -1.4, -√5 (but not √-1, which is an imaginary or complex number)

________ (Gaussian) ________: a continuous probability distribution in probability theory and statistics that describes data which clusters around the mean in a curved "bell curve", highest in the middle and quickly tapering off to each side

________ ________: a line on which all points correspond to real numbers (a simple number line may only mark integers, but in theory all real numbers to +/- infinity can be shown on a number line)

________ ________: the branch of pure mathematics concerned with the properties of numbers in general, and integers in particular

O

________ ________: an extension of the natural numbers (different from integers and from cardinal numbers) used to describe the order type of sets i.e. the order of elements within a set or series

P

________: a type of conic section curve, any point of which is equally distant from a fixed focus point and a fixed straight line

________: a statement that appears to contradict itself, suggesting a solution which is actually impossible

________: a mathematical term that describes the property of an integer's inclusion in one of two categories: even or odd, e.g. an integer is even if it is 'evenly divisible' by two (the old-fashioned term "evenly divisible" is now almost always shortened to "divisible") and odd if it is not even (6 is even because there is no remainder when dividing it by 2)

________ ________ ________: a relation involving an unknown function with several independent variables and its partial derivatives with respect to those variables

________ ________: a number that is the sum of its divisors (excluding the number itself), e.g. 28 = 1 + 2 + 4 + 7 + 14

________ ________: a function that repeats its values in regular intervals or periods, such as the trigonometric functions of sine, cosine, tangent, etc

________: a particular ordering of a set of objects, e.g. given the set {1, 2, 3}, there are six permutations: {1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, and {3, 2, 1}

________: the ratio of a circumference of a circle to its diameter, an irrational (and transcendental) number approximately equal to 3.141593...

________: positional notation for numbers, allowing the use of the same symbols for different orders of magnitude, e.g. the "one's place", "ten's place", "hundred's place", etc.

________ ________: a two-dimensional coordinate system in which each point on a plane is determined by its distance r from a fixed point (e.g. the origin) and its angle θ (theta) from a fixed direction (e.g the x axis)

________: an algebraic expression or equation with more than one term, constructed from variables and constants using only the operations of addition, subtraction, multiplication and non-negative whole-number exponents, e.g. $5x^2 - 4x + 4y + 7$

________: a statement that is taken to be true, to serve as a premise or starting point for further reasoning and arguments

________ ________: integers greater than 1 which are only divisible by themselves and 1

________ ________: the branch of mathematics concerned with analysis of random variables and events, and with the interpretation of probabilities (the likelihood of an event happening)

Q

________ ________: a polynomial equation with a degree of 2 (i.e. the highest power is 2), of the form $ax^2 + bx + c = 0$, which can be solved by various methods including factoring, completing the square, graphing, Newton's method and the quadratic formula

________: the act of squaring, or finding a square equal in area to a given figure, or finding the area of a geometrical figure or the area under a curve (such as by a process of numerical integration)

________ ________: a polynomial having a degree of 4 (i.e. the highest power is 4), of the form $ax^4 + bx^3 + cx^2 + dx + e = 0$, the highest order polynomial equation that can be solved by factorization into radicals by a general formula

________: a number system that extends complex numbers to four dimensions (so that an object is described by a real number and three complex numbers, all mutually perpendicular to each other), which can be used to represent a three-dimensional rotation by just an angle and a vector

________ ________: a polynomial having a degree of 5 (i.e. the highest power is 5), of the form $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$, not solvable by factorization into radicals for all rational numbers

R

________ ________: numbers that can be expressed as a fraction (or ratio) $^{a}/_{b}$ of two integers (the integers are therefore a subset of the rationals), or alternatively a decimal which terminates after a finite number of digits or begins to repeat a sequence

________ ________: all numbers (including natural numbers, integers, decimals, rational numbers and irrational numbers) which do not involve imaginary numbers (multiples of the imaginary unit *i*, or the square root of -1), may be thought of as all points on an infinitely long number line

________: a number which, when multiplied by *x* yields the multiplicative identity 1, and can therefore be thought of as the inverse of multiplication, e.g. the reciprocal of *x* is $^{1}/_{x}$, the reciprocal of $^{3}/_{5}$ is $^{5}/_{3}$

________ ________: a triangle (three sided polygon) containing an angle of 90°

S

______-_______ object is exactly or approximately similar to a part of itself (in fractals, the shapes of lines at different iterations look like smaller versions of earlier shapes)

________: an ordered set whose elements are usually determined based on some function of the counting numbers, e.g. a geometric sequence is a set where each element is a multiple of the previous element; an arithmetic sequence is a set where each element is the previous element plus or minus a number

________: a collection of distinct objects or numbers, without regard to their order, considered as an object in its own right

________ ________: the number of digits to consider when using measuring numbers, those digits that carry meaning contributing to its precision (i.e. ignoring leading and trailing zeros)

________ ________: a set or system of equations containing multiple variables which has a solution that simultaneously satisfies all of the equations (e.g. the set of simultaneous linear equations $2x + y = 8$ and $x + y = 6$, has a solution $x = 2$ and $y = 4$)

________ ________: in mathematics, a square root of a number *a* is a number *y* such that $y^2 = a$; in other words, a number *y* whose *square* (the result of multiplying the number by itself, or $y \times y$) is *a*

________: a subsidiary collection of objects that all belong to, or is contained in, an original given set, e.g. subsets of $\{a, b\}$ could include: $\{a\}$, $\{b\}$, $\{a, b\}$ and $\{\}$

________: the n-th root a number, such as $\sqrt{5}$, the cube root of 7, etc

________: the correspondence in size, form or arrangement of parts on a plane or line (line symmetry is where each point on one side of a line has a corresponding point on the opposite side, e.g. a picture a butterfly with wings that are identical on either side; plane symmetry refers to similar figures being repeated at different but regular locations on the plane)

T

________: a collection of numbers at every point in space which describe how much the space is curved, e.g. in four spatial dimensions, a collection of ten numbers is needed at each point to describe the properties of the mathematical space or manifold, no matter how distorted it may be

________: in an algebraic expression or equation, either a single number or variable, or the product of several numbers and variables separated from another term by a + or - sign, e.g. in the expression $3 + 4x + 5yzw$, the 3, the $4x$ and the $5yzw$ are all separate terms

________: a mathematical statement or hypothesis which has been proved on the basis of previously established theorems and previously accepted axioms, effectively the proof of the truth of a statement or expression

________: the field of mathematics concerned with spatial properties that are preserved under continuous deformations of objects (such as stretching, bending and morphing, but not tearing or gluing)

________ ________: an irrational number that is "not algebraic", i.e. no finite sequence of algebraic operations on integers (such as powers, roots, sums, etc.) can be equal to its value, examples being π and e. For example, $\sqrt{2}$ is irrational but not transcendental because it is the solution to the polynomial $x^2 = 2$.

________ ________: cardinal numbers or ordinal numbers that are larger than all finite numbers, yet not necessarily absolutely infinite

________ ________: a number which can be represented as an equilateral triangle of dots, and is the sum of all the consecutive numbers up to its largest prime factor - it can also be calculated as $^{n(n+1)}/_2$, e.g. $15 = 1 + 2 + 3 + 4 + 5 = {}^{5(5+1)}/_2$

________: a polygon with three edges and three vertices, e.g. a triangle with vertices A, B, and C is denoted ΔABC

________: an algebraic equation with 3 terms, e.g. $3x + 5y + 8z$; $3x^3 + 2x^2 + x$; etc

________ ________: an alternative to naive set theory in which all mathematical entities are assigned to a type within a hierarchy of types, so that objects of a given type are built exclusively from objects of preceding types lower in the hierarchy, thus preventing loops and paradoxes

V

________: a physical quantity having magnitude and direction, represented by a directed arrow indicating its orientation in space

________ ________: a three-dimensional area where vectors can be plotted, or a mathematical structure formed by a collection of vectors

Attachment I

Writing a summary

A **summary** is a shorter version of the original text. Such a simplification highlights the major points from the much longer subject, such as a text, speech, film, or event. In order to write a good summary, use your own words to express briefly the main idea and relevant details of the piece you have read. Your purpose in writing the summary is to give the basic ideas of the original reading.

5 main steps in writing summary:

1. Look through the text and try to divide the text into parts. Determine what type of text you are dealing with. Don't take any notes - just read.

2. Read the text, highlight important information and take notes.

3. Write down the main points of each part, in your own words (this will make it easier to write later).

4. Write down the key support points for the main topic, but do not include minor detail.

5. Go through the text again, make changes as appropriate.

Useful language

Introduction

(Title) is a novel by (author).

(Title) was written by (author).

The story is about (topic).

The novel tells the story of (hero/topic).

(Title) tells of (hero), who ...

In (title) by (author), the reader is taken into (place/time of story).

(Title) is the story of (hero/action/...)

(Title) is set in the period of (event).

The text presents/describes…

Content

As the story begins, ...

During ...

While ...

As/When ...

Since/As ...

Just then ...

After ...

Before ...

Before long ...

Soon ...

Soon afterwards ...

As soon as ...

One day/evening ...

The following day ...

Some time later ...

Hours/Months/Years later, ...

By morning/the next day/the time ...

Meanwhile ...

However, ...

Again/Once again ...

At this point ...

To his surprise ...

This incident is/was followed by ...

To make matters even worse ...

Eventually, .../Finally, ...

<u>The author</u>

Says, states, points out that…

Claims, thinks, believes that…

Describes, explains, makes clear that…

Criticizes, analyses, comments on…

Tries to express…

Argues that…

Suggests that…

Compares X to Y…

Doubts that…

Tries to convince the readers that…

Concludes that…