Transformation of the frequency-modulated continuous-wave field into a train of short pulses by resonant filters

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Abstract

The resonant filtering method transforming frequency modulated radiation field into a train of short pulses is proposed to be applied in optical domain. Effective frequency modulation can be achieved by electro-optic modulator. Due to frequency modulation narrow-spectrum CW radiation field is seen by the resonant filter as a comb of equidistant spectral components separated by the modulation frequency. Tuning narrow-bandwidth filter in resonance with n-th spectral component of the comb transforms the radiation field into bunches of pulses with n pulses in each bunch. The transformation is explained by the interference of the coherently scattered resonant component of the field with the whole comb. Constructive interference results in formation of pulses, while destructive interference is seen as dark windows between pulses. It is indicated that the optimal thickness of the resonant filter is several orders of magnitude smaller than the necessary thickness of the dispersive filters used before in optical domain to produce short pulses from the frequency modulated field.

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I. INTRODUCTION

Generating pedestal-free optical pulses with high peak power from a low-power laser is of great interest in optical communication [1]. Existing devices generally employ electro-optic amplitude modulators [2], acousto-optic modulators [3–5], frequency chirping followed by dispersive compensators [6–10], and dispersive modulators [11, 12]. Application of the rapid π-phase-shift technique for CW radiation field with subsequent filtering by the optically thick resonant absorber is also capable to produce short pulses in a controllable way [13–19]. However, this technique demands very fast phase switch, otherwise the amplitudes of the pulses reduce appreciably.

Recently, original technique for producing short pulses was reported in [20, 21]. This technique was experimentally tested with gamma photons having long coherence length (long duration of a single-photon wave packet) and the method of splitting of a single photon into pulses [20, 21] was proposed to create time-bin qubits, whose concept was introduced before in [22, 23] for optical photons. This technique could be also practical for protocols of memory-based quantum networks [24–27], which require narrow-band photons with optimal temporal waveforms to interact with atoms efficiently.

The method of single photon shaping [20, 21] is based on frequency modulation of the radiation field, which is also one of the basic elements of the frequency chirping, implemented in [7, 8] by electro-optic modulator. However, in spite of following dispersion compensator, which is accomplished in [7, 8] by near resonance absorber containing alkaline vapor, subsequent absorption (removal) of a particular spectral component is used [20, 21]. This removal method is much more flexible compared with the frequency chirping followed by a dispersive compensators [7, 8] and allows fine control of the duration and repetition rate of the pulses.

In this paper we analyze the capabilities of the removal method for application in optical domain and compare it with dispersion compensator method. The removal method could be implemented with cold atoms, atomic vapors, and organic molecules doped in polymer matrix. They could serve as filters to remove single frequency component of the comb spectrum produced by electro-optical modulator from CW radiation field. Cold atoms possess almost naturally broadened Lorentzian absorption lines while atomic vapors at low pressure demonstrate the Doppler-broadened absorption lines. The effect of the wings of these lines on the shape of the pulses is discussed. The proposed method is capable to produce

nanosecond or subnanonesocond pulses, which could be directly resolved in time by modern detectors with no use of cross-correlation technique, based on Mach-Zehnder interferometer with a delay line in one of the arms.

There are also proposals and experimental implementations producing short pulses by strong laser excitation of atoms [28–32] and molecules [33–37]. The proposals [28–32] consider time-dependent perturbation of the excited atomic energy level by the strong far off-resonance laser due to Stark effect. The method [33–37] is based on the stimulated Raman scattering of a strong bichromatic laser radiation, which produces large molecular coherence employing electromagnetically induced transparency scheme.

In this paper the techniques based on using strong radiation fields are not considered, since the main focus of interest is the transformation of very weak fields into pulses with controllable waveforms. High efficiency and low losses of the transformation are quite important from the viewpoint of applications in telecommunication.

The paper is organized as follows. In Sec. II, a general formalism of the resonant filtering of phase modulated field is presented. In Sec. III, the resonant filtering through laser-cooled atoms is discussed. In Sec. IV, atomic vapor of hot atoms is considered as a resonant filter. In Sec. V, creation of femtosecond pulses by filtering high order harmonics of the phase modulated field through atomic vapor is proposed. In Sec. VI, comparison of the resonant filtering with dispersive filters is discussed.

II. BASIC IDEA

CW radiation field $E(t) = E_0 \exp(-i\omega_r t + ikz)$ after passing through the electro-optic modulator acquires phase modulation

$$E_{EO}(t) = E(t)e^{im\sin\Omega t}, \tag{1}$$

where Ω and m are the frequency and index of phase modulation. According to Jacobi-Anger expansion

$$E_{EO}(t) = E(t)\sum_{n=-\infty}^{+\infty} J_n(m)e^{im\Omega t}, \tag{2}$$

this field is transformed into an equidistant frequency comb with spectral components $\omega_n = \omega_r - n\Omega$, where $J_n(m)$ is the Bessel function of the n-th order. Fourier transform of the field

is

$$E_{EO}(\omega) = E_0 \sum_{n=-\infty}^{+\infty} J_n(m)\delta(\omega - \omega_n), \tag{3}$$

where $\delta(x)$ is the Dirac delta function. If CW radiation field has finite spectral width, then δ function is to be substituted by $f_r(\omega - \omega_n)$ describing the spectrum of the CW field.

We transmit the frequency comb through the resonant filter with a single absorption line $F(\omega - \omega_f)$ centered at frequency ω_f. We select the filter whose absorption linewidth Γ_f is much smaller than the distance Ω between neighboring components of the frequency comb. Such a filter is capable to remove selectively one of the spectral components of the comb. Below we don't show explicit spatial dependence of the field amplitude hiding it into parameters of the filtered field.

By changing the carrier frequency of the CW radiation field we tune the n-th component of the comb $\omega_n = \omega_r - n\Omega$ close to resonance with the filter frequency ω_f. Then the radiation field at the exit of the filter is transformed as

$$E_{fnA}(\omega) = E_{EO}(\omega) + E_0 J_n(m)\left[T(\Delta_n) - 1\right]\delta(\omega - \omega_n), \tag{4}$$

where $\Delta_n = \omega_n - \omega_f$ and $T(\Delta_n)$ is a transmission function of the filter, which depends on its absorption coefficient and physical thickness (see Sec. III). In general, the transmission function $T(\Delta_n)$ is a complex function, which takes into account attenuation of the field amplitude and phase shift due to the frequency dependent refraction index (dispersion) after passing through the filter of length L.

For the optically thick filter at exact resonance $T(0) = T_0$ tends to zero. Indexes n and A in $E_{fnA}(\omega)$ mean that n-th component of the comb is in resonance with the filter and solution is approximate since in Eq. (4) we disregard the interaction of nonresonant components with the filter. Small contribution from these components due to dispersion will be taken into account in Sec. III.

Equation (4) has a simple physical meaning. It is constructed such that the first term in square brackets just describes the amplitude of the attenuated spectral component, which is in resonance and proportional to $J_n(m)T_0$. The second term removes from the comb $E_{EO}(\omega)$ the resonant component to avoid taking it into account twice. Within this approximation, other spectral components pass through the resonant filter with no change.

Inverse Fourier transformation of eq. (4) results in

$$E_{fnA}(t) = E_0 e^{-i\omega_r t}\left[e^{im\sin\Omega t} + (T_0 - 1)J_n(m)e^{in\Omega t}\right]. \tag{5}$$

The interpretation of this result is based on the interference of the incident and coherently scattered radiation fields [21]. The latter is described by the second term in the square brackets in Eq. (5). Therefore, for the frequency comb whose n-th spectral component interacts with the filter, just this component is coherently scattered by the atoms in the filter. The scattered field interferes with the whole frequency comb at the exit of the filter. Therefore the output radiation field reveals unusual properties.

The intensity of the field at the exit of the filter $I_{fnA}(t) = |E_{fnA}(t)|^2$ is described by equation

$$I_{fnA}(t) = I_0 \left[1 - 2S_n \cos\psi_n(t) + S_n^2\right], \tag{6}$$

where $I_0 = |E_0|^2$, $S_n = (1 - T_0)J_n(m)$, and $\psi_n(t) = n\Omega t - m\sin\Omega t$. If the filter is opaque for the resonant component, the amplitude of the scattered field almost coincides with the amplitude of the resonant component and $S_n \to J_n(m)$. The phases of these fields are opposite, therefore, if $\cos\psi_n(t)$ is positive we have destructive interference seen as a drop of intensity $I_{fnA}(t)$. If $\cos\psi_n(t)$ is negative, the comb and the scattered resonant component interfere constructively producing pulses. The interference becomes pronounced if $J_n(m)$ has global maximum. For different spectral components n this maximum is achieved at different values of the modulation index m. We denote these values as m_n, which are $m_1 = 1.8$, $m_2 = 3.1$, $m_3 = 4.2$, ... The phase difference of the comb $E_{EO}(t)$ (whose phase evolves as $m\sin\Omega t$) and the scattered field $E_{sct}(t)$ (whose phase evolves as $n\Omega t$) is $\psi_n(t) + \pi$ if $J_n(m) > 0$. The evolution of $\psi_n(t)$ in time fully describes the formation of the pulses and the dark windows between them.

The phase modulated field $E_{EO}(t)$ after passing through the resonant filter is transformed into bunches of pulses. The number of pulses in a bunch is equal to the number of filtered component of the frequency comb n. The intensity of the pulses is equal to $[1+S_n(t)]^2 I_0$. For example, the maximum intensities, predicted by Eq. (6) for optimal values of the modulation index m_n and opaque filter ($T_0 = 0$), are $I_{\max} = 2.5I_0$ for $n = 1$, $I_{\max} = 2.1I_0$ for $n = 2$, and $I_{\max} = 2.06I_0$ for $n = 3$. Thus, the intensity of the pulses exceeds almost two times the intensity of the radiation field if it would not interact with the filter. The radiation intensity between bunches is quite small because of destructive interference, which predicts $I_{\min} = [1 - S_n(t)]^2 I_0$. For the same values of the parameters T_0 and m_n this intensity is $I_{\min} = 0.175I_0$ for $n = 1$, $I_{\min} = 0.26I_0$ for $n = 2$, and $I_{\min} = 0.32I_0$ for $n = 3$, i.e., almost an order of magnitude smaller compared with the pulse maxima.

Within the period of the phase oscillation ($T_{EO} = 2\pi/\Omega$), induced by electro-optical modulator, one can distinguish two intervals. Half of the period $T_{EO}/2$ the phase $\psi_n(t)$ evolves almost linearly as $(n+m)\Omega t + C$ where C is constant but different for different bunches. Within this half of the period the relation $\psi_n(t) = (2k+1)\pi$, where k is integer, is satisfied n-times, producing pulses. During the other half of the period $T_{EO}/2$ the evolution of phase $\psi_n(t)$ almost stops near the value $2k\pi$, resulting in destructive interference, seen as dark windows.

III. FILTERING TROUGH COLD ATOMS

In this section we consider the frequency comb filtering through laser-cooled atoms with a modest optical depth. As an example we take parameters of ^{85}Rb atoms in a two-dimensional magneto-optical trap, described, for example, in [13]. CW radiation field excites ^{85}Rb D1-line transition ($\lambda = 795$ nm). Since the absorption line is almost Lorentzian, the transmission function in Eq. (4) can be described as [38]

$$T(\Delta_n) = \exp\left(-\frac{\alpha L\gamma/2}{\gamma - i\Delta_n}\right), \tag{7}$$

where α is the Beer's law absorption coefficient, L is the length of atomic cloud, and γ is a halfwidth of absorption line. If n-th component of the frequency comb is in exact resonance with D1 line, then the transmission function is $T(0) = \exp(-\alpha L/2)$. Atomic cloud with a length of few millimeters demonstrates already optical depth $\alpha L = 5$.

One can verify the approximate expression for the filtered field $E_{fnA}(t)$ (5) comparing it with the exact expression, which could be obtained by the convolution of $E_{EO}(t)$ field

$$E_{fn}(t) = \int_{-\infty}^{+\infty} E_{EO}(t-\tau)R(\tau)d\tau, \tag{8}$$

with the Green's function of the absorber of thickness L [16, 17, 38]

$$R(t) = \delta(t) - \Theta(t)e^{-(i\omega_f+\gamma)t}j_1(bt) \tag{9}$$

where $\delta(t)$ is the Dirac δ function, $\Theta(t)$ is the Heaviside step function, $j_1(bt) = \sqrt{b/t}J_1\left(2\sqrt{bt}\right)$, $J_1(x)$ is the Bessel function of the first order, and $b = \alpha L\gamma/2$. For the

infinitely lasting field, Eq. (8) is reduced to

$$E_{fn}(t) = E_{EO}(t) - E(t)\int_0^{+\infty} j_1(b\tau)e^{(i\Delta-\gamma)\tau+im\sin\Omega(t-\tau)}d\tau. \quad (10)$$

where $\Delta = \omega_r - \omega_f$. Index n in $E_{fn}(t)$ implies that we consider the case when n-th spectral component of the comb is close to resonance with the filter, i.e., $\Delta = n\Omega + \Delta_n$ and Δ_n is close to zero.

Equation (10) takes into account the transformation of all spectral components of the comb including those whose change is infinitely small since they are far from resonance with the filter. Comparison of the time dependencies of the approximate intensity $I_{fnA}(t)$, Eq. (6), and exact expression $I_{fn}(t) = |E_{fn}(t)|^2$, where $\Delta = n\Omega$ and $\Delta_n = 0$, is shown in Fig. 1 for $n = 1$, 2, and 3. Optimal values of the modulation index $m = m_n$ are adopted in each case. The following parameters for the cold-atom filter: $\gamma/2\pi = 3$ MHz, $\alpha L = 5$, and $b/2\pi = 7.5$ MHz, are taken. We choose the modulation frequency of the field phase equal to 30 MHz, satisfying well the condition that Ω is much larger than the spectral width of absorption line of the filter $\Gamma_f = 2\gamma$.

In Fig. 1(a) pulse duration from shoulder to shoulder for $n = 1$ is slightly shorter than half of the period of the phase oscillation $T_{EO}/2 = 16.7$ ns, while duration of the dark window is slightly longer than $T_{EO}/2$. The pulsewidth at halfmaximum is close to but slightly shorter than $T_{EO}/4 = 8.3$ ns. If we have $n > 1$ pulses in a bunch (see Fig. 1), their pulsewidth can be roughly estimated as $T_{EO}/4n$. Therefore, if, for example, $n = 3$, the pulsewidth is already close to 1 ns.

Small misfit between exact and approximate time dependencies is caused by the contribution of two nearest neighbors $\omega_{n\pm1} = \omega_r - (n \pm 1)\Omega$ of the resonant spectral component ω_n. The corrected expression, which takes them into account, is easily found

$$E_{fn}(t) \approx E_{fnA}(t) + E_{n+1}(t) + E_{n-1}(t), \quad (11)$$

where

$$E_{n\pm1}(t) = E_0 e^{-i\omega_r t + i(n\pm1)\Omega t} J_{n\pm1}(m_n)\left(e^{\frac{-b}{\gamma - i(\Delta_n \mp \Omega)}} - 1\right). \quad (12)$$

The field intensity, calculated with this correction, describes almost excellent the exact time dependence of the filtered field intensity $I_{fn}(t)$. As it is seen in Fig.1, the contribution of the sidebands introduces the asymmetry of the pulse intensities within a bunch and after-ringing, which appears in the beginning of dark windows.

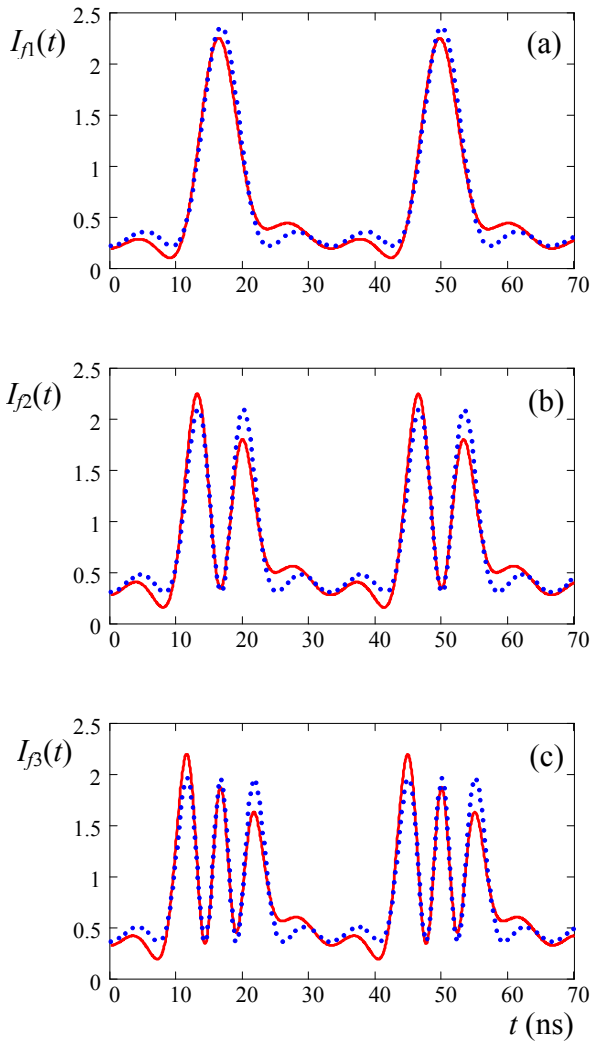

FIG. 1: (color on line) Time dependence of the intensity of the filtered radiation field I_{fn}, where n means the number of the spectral component tuned in resonance with the filter. Solid line in red corresponds to exact expression (10) and dotted line in blue represents the analytical approximation (6). Both are normalized to the intensity of the incident radiation field I_0. The modulation frequency $\Omega/2\pi = 30$ MHz is ten times larger than the halfwidth of the absorption line of the filter $\gamma/2\pi = 3$ MHz, the optical depth of the filter is $\alpha_0 L = 5$. The number of the spectral component n, tuned in resonance ($\Delta_n = 0$), and the optimal values of the modulation index are $n = 1$, $m_1 = 1.8$ in (a), $n = 2$, $m_2 = 3.1$ in (b), and $n = 3$ $m_3 = 4.2$ in (c).

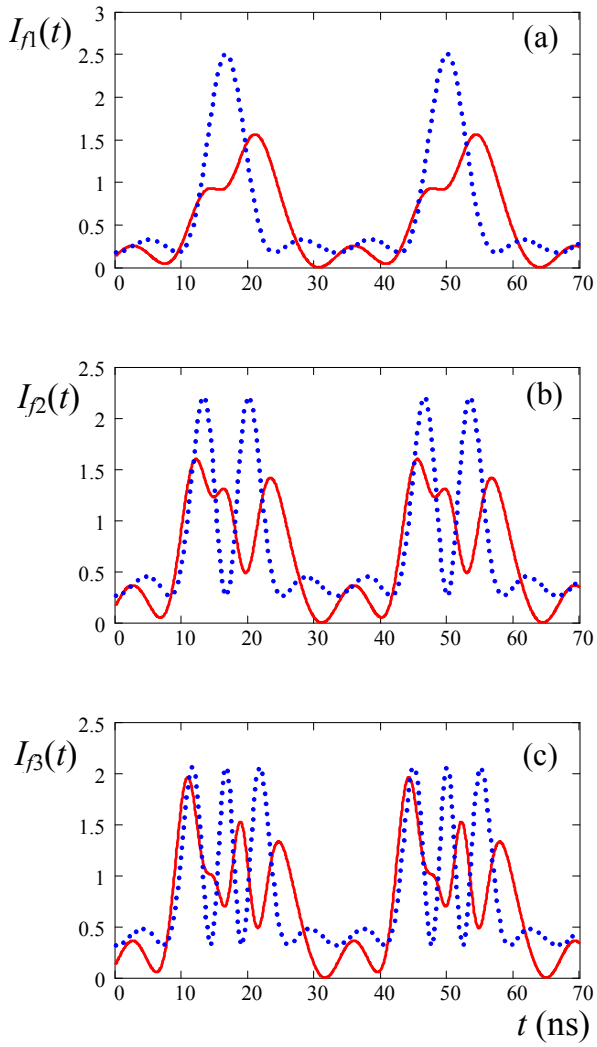

FIG. 2: Time dependence of the intensity of the filtered radiation field for atomic cloud of the length $L = 1.5$ cm, which corresponds to the optical depth $\alpha_0 L = 33$ [13]. Other parameters and notations are the same as in Fig. 1

Parameter $b = \alpha L\gamma/2$ plays a crucial role in the radiation filtering since the transmission function $T(\Delta) = \exp\left[-b/(\gamma - i\Delta)\right]$ essentially broadens if b becomes larger than γ. In this case many spectral components of the comb are modified by the filter and the interference of the comb with the scattered radiation field becomes messy. Figure 2 shows a comparison of the time dependencies of exact intensity $I_{fn}(t)$ and approximate one $I_{fnA}(t)$ for atomic cloud with optical depth $\alpha L = 33$, which is achieved in [13] with cloud length $L = 1.5$ cm. Instead of nice pulses we see their appreciable distortion. This is not surprising since for this cloud the parameter $b/2\pi = 49.5$ MHz is larger than the frequency comb spacing $\Omega/2\pi = 30$ MHz

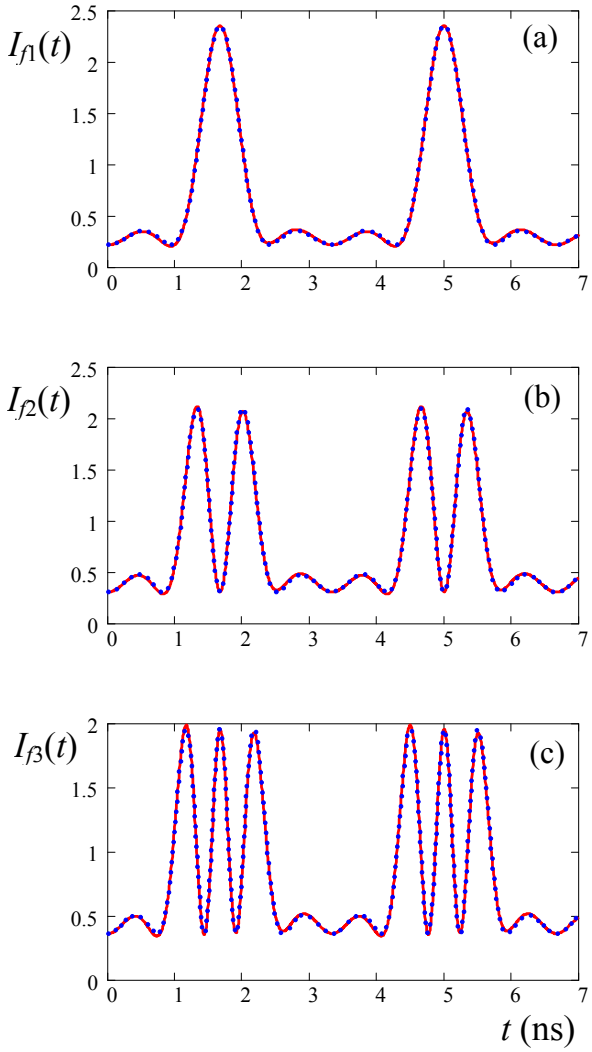

FIG. 3: Comparison of the exact time dependence of the intensity of the filtered radiation field $I_{fn}(t)$ (solid line in red) with the approximate one $I_{fnA}(t)$ (blue dots) for atomic cloud with optical depth $\alpha_0 L = 5$. Modulation frequency is $\Omega/2\pi = 300$ MHz. Other parameters and notations are the same as in Fig. 1

and many sidebands of the resonant component contribute to the pulse generating. For these values of the parameters it is necessary to take 8 neighboring components into account, i.e., four red detuned from resonance and four blue detuned. Then, the approximate expression similar to Eq. (11) but with eight additional terms $E_{n\pm k}(t)$, where $k = 1$, 2, 3, and 4, gives the same result as the exact equation (10). Actually the exact equation can be expressed as a result of filtering of all spectral components of the comb

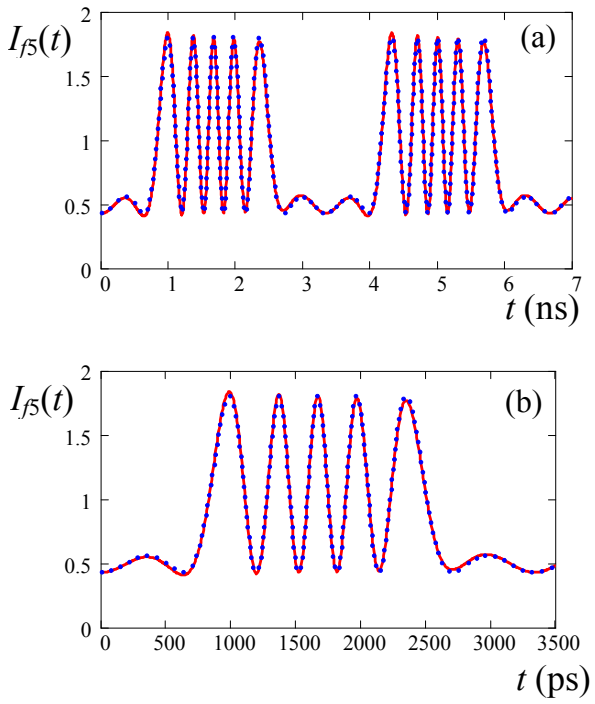

FIG. 4: (a) Time evolution of intensity of the filtered radiation field for atomic cloud with optical depth $\alpha_0 L = 5$. Modulation frequency is $\Omega/2\pi = 300$ MHz. The sideband $n = 5$ is tuned in resonance with the filter. The value of the modulation index $m_5 = 6.4$ is close to 2π. (b) Zoom in the first bunch of pules, shown in (a). Solid line in red shows $I_{fn}(t)$ and blue dots correspond to $I_{fnA}(t)$. Both are normalized to I_0.

$$E_{fn}(t) = E_{fnA}(t) + \sum_{k=1}^{\infty} \left[E_{n+k}(t) + E_{n-k}(t) \right], \tag{13}$$

where

$$E_{n\pm k}(t) = E_0 e^{-i\omega_r t + i(n\pm k)\Omega t} J_{n\pm k}(m_n) \left(e^{\frac{-b}{\gamma - i(\Delta_n \mp k\Omega)}} - 1 \right). \tag{14}$$

Contrary to the example of the optically thick filter modifying many spectral components of the comb, we give another example when filtering becomes ideal. We take moderate optical depth $\alpha L = 5$, which corresponds to $b/2\pi = 7.5$ MHz, and increase modulation frequency ten times to the value $\Omega/2\pi = 300$ MHz. Then, the contribution of the sidebands becomes negligible and the intensity of the filtered radiation is well described by approximate equation (6) (see Fig. 3).

It is interesting to note that the phase modulation with frequency 300 MHz is capable

to produce subnanosecond pulses. Tuning, for example, the 5-th sideband of the frequency comb into resonance with the filter is capable to produce pulses with duration of 167 ps (see Fig. 4).

Cold ^{85}Rb atoms are not the only example of the narrow bandwidth filter. One can use also a cloud of cold potassium (^{39}K) atoms generated in a vapor-cell magneto optic trap whose excitation on the $4S_{1/2}(F = 1) \leftrightarrow 4P_{1/2}(F = 2)$ transition (transition wave-length 770 nm) was studied in [39] for observation of optical precursors.

IV. FILTERING TROUGH ATOMIC VAPOR

In this section we consider the filtering of the frequency comb through a vapor of ^{87}Rb atoms and take the parameters of the experiment [40] where spectral properties of the electromagnetically induced transparency were studied. Assume that the fundamental frequency of the comb is close to the $S_{1/2}, F = 1 \rightarrow P_{1/2}, F = 2$ transition of the D_1 line of natural Rb ($\lambda = 795$ nm). The atoms are confined in a cell of length $L = 5$ cm. We take two temperatures of Rb vapor (50 and 70°C), which correspond to atomic densities $N_1 = 6 \times 10^{10}$ cm^{-3} and $N_2 = 6 \times 10^{11}$ cm^{-3}, respectively. Natural linewidth of the Rb D_1 line is $\Gamma_f/2\pi = \gamma/\pi = 5.4$ MHz and Doppler broadening is $\Delta\omega_D/2\pi = 500$ MHz. We take the phase modulation frequency $\Omega/2\pi = 10$ GHz, which is 20 times larger than the Doppler width $\Delta\omega_D/2\pi = 500$ MHz.

The transmission function for the atomic vapor is

$$T_D(\Delta_n) = \exp[-\alpha_{1,2} L F_D(\Delta_n)/2], \tag{15}$$

where $\alpha_{1,2} = 3N_{1,2}\lambda^2/2\pi$ is the absorption coefficient of the naturally broadened line and $F_D(\Delta_n)$ is a Doppler broadened absorption line, which is

$$F_D(\Delta_n) = \sqrt{\frac{\ln 2}{\pi}} \frac{2\gamma}{\Delta\omega_D} \int_{-\infty}^{+\infty} \frac{\exp[-\ln 2(2x/\Delta\omega_D)^2]}{\gamma - i(\Delta_n + x)} dx. \tag{16}$$

We have to emphasize that for both densities of Rb atoms the cell is optically thick, i.e., $\alpha_1 L = 905$ and $\alpha_2 L = 9053$. It is easy to show (see, for example, [41]) that at exact resonance ($\Delta_n = 0$) Eq. (16) can be approximated as

$$F_D(0) = \sqrt{\pi \ln 2} \frac{2\gamma}{\Delta\omega_D}. \tag{17}$$

Therefore for $\Delta_n = 0$ the effective optical depth, $\alpha_{1,2}LF_D(0)$, is reduced almost hundred times since the Doppler width $\Delta\omega_D$ is two orders of magnitude larger than the natural linewidth 2γ.

If $|\Delta_n| > 1.8\Delta\omega_D$, then the absorption line reveals Lorenzian wings (see, for example, Fig. 1 in Ref. [41]), which can be approximated as

$$F_D(\Delta_n) = \frac{\gamma}{\gamma - i\Delta_n}. \tag{18}$$

Therefore, the contribution of far wings of the dispersion $\chi'(\Delta_{n\pm k}) \sim \mathrm{Im}\, F_D(\Delta_{n\pm k})$ in the filtering of nonresonant components $\omega_r - (n \pm k)\Omega$ of the frequency comb could be noticeable if $\Delta_n = 0$. For example, the contribution of the nearest sidebands ($k = \pm 1$) of the resonant component is proportional to

$$E_{n\pm1}(t) \sim J_{n\pm1}(m_n)\left(e^{\mp ib_{1,2}/\Omega} - 1\right), \tag{19}$$

where $b_{1,2} = \alpha_{1,2}\gamma/2$. For the atomic density N_1 the ratio $b_1/\Omega = 0.122$ is small since $b_1/2\pi = 1.2$ GHz and modification of the sidebands due to filtering does not influence significantly the shape of the produced pulses. For the atomic density N_2 the ratio $b_2/\Omega = 1.22$ is large since $b_2/2\pi = 12$ GHz and the sidebands change their phases due to filtering. In this case one can expect appreciable corruption of the produced pulses. To verify these expectations we calculate the intensity of the filtered comb taking into account the modification of many sidebands. The number of them can be limited by $\pm k_{\max}$ if the contribution of the next sidebands $\pm(k_{\max} + 1)$ does not change the signal.

Substituting the transmission function $T_D(\Delta_n)$ into equations (5) and (14) one obtains the modified Eq. (13), which describes the transformation of the frequency comb after passing through the atomic-vapor filter. The substitution changes the functions S_n and $E_{n\pm k}(t)$ to

$$S_n = J_n(m_n)[1 - T_D(0)], \tag{20}$$

$$E_{n\pm k}(t) = E_0 e^{-i\omega_r t + i(n\pm k)\Omega t} J_{n\pm k}(m_n)\left[T_D(\mp k\Omega) - 1\right], \tag{21}$$

where $\Delta_n = 0$, which implies that the n-th spectral component is in exact resonance with the filter.

For the Rb cell with atomic density $N_1 = 6 \times 10^{10}$ cm^{-3} it is enough to take into account the contribution of two spectral components $(n \pm 1)$ neighboring the resonant component n. For this density the effective optical depth of the cell at the absorption line center is

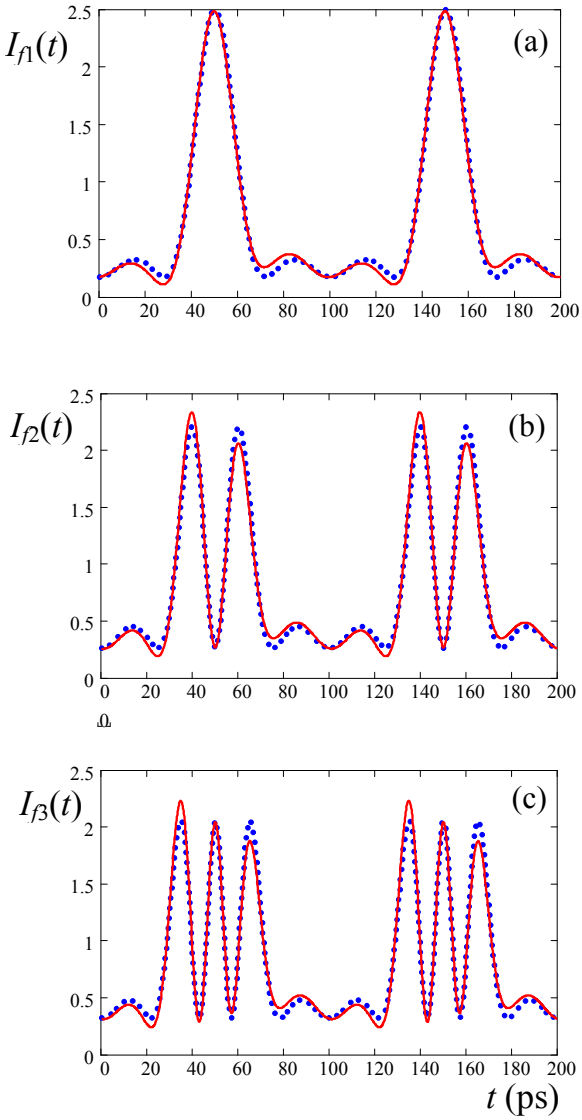

FIG. 5: Time dependence of the intensity of the filtered radiation field $I_{fn}(t)$ (solid line in red) for atomic vapor with optical depth $\alpha_1 L = 905$. Modulation frequency is $\Omega/2\pi/2\pi = 10$ GHz. The approximate time dependence of the intensity $I_{fnA}(t)$ is shown by blue dots. Other parameters and notations are the same as in Fig. 1

$\alpha_1 L F_D(0) = 14.4$. The result of the comb filtering through the cell is shown in Fig. 5. If $n = 1$, 2, or 3 spectral component is in resonance with the filter, the pulses with the width 25, 12.5, and 8.3 ps, respectively, are produced.

If atomic density is increased by the order of magnitude to $N_2 = 6 \times 10^{11}$ cm^{-3}, then eight sidebands of the resonant component $n \pm 1$, $n \pm 2$, $n \pm 3$, and $n \pm 4$ give noticeable

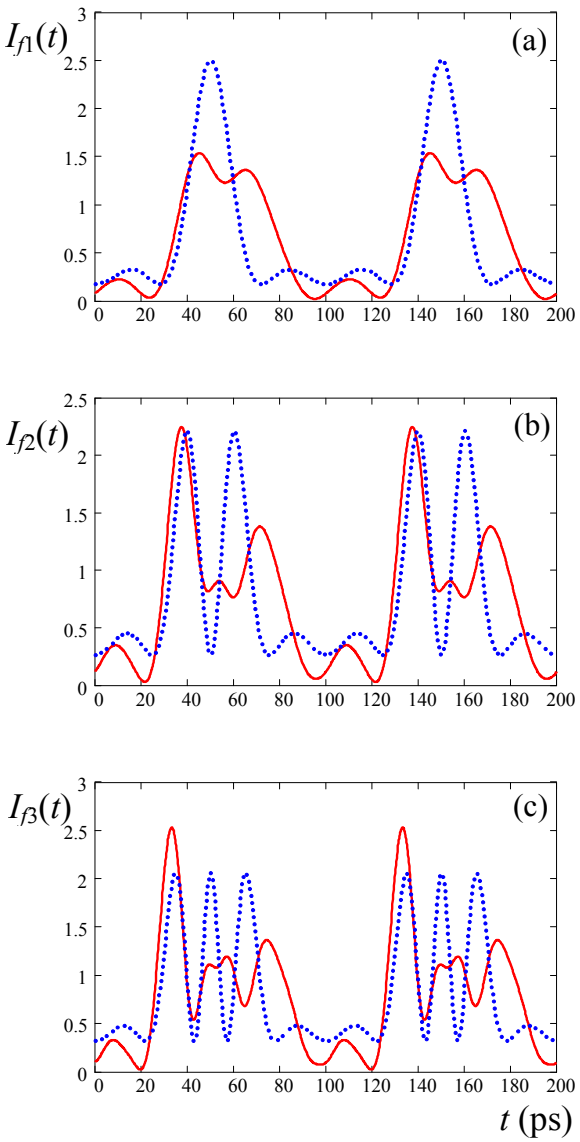

FIG. 6: Time dependence of intensity of the radiation field filtered through the atomic vapor with the optical depth $\alpha_2 L = 9053$ (solid line in red). Other parameters and notations are the same as in Fig. 5

contribution. Therefore, the intensity modulation of the filtered comb becomes messy (see Fig. 6). Actually, for $n = 2$ and $n = 3$ it is necessary to take into account the contribution of 14 sidebands (up to $n \pm 7$).

To obtain nice and clean pulses it is preferable to use a filter with a smaller optical depth. An example of filtering by a cell with optical depth $\alpha L = 453$ is shown in Fig. 7. The filter is tuned in resonance with the spectral component $n = 10$ of the frequency comb,

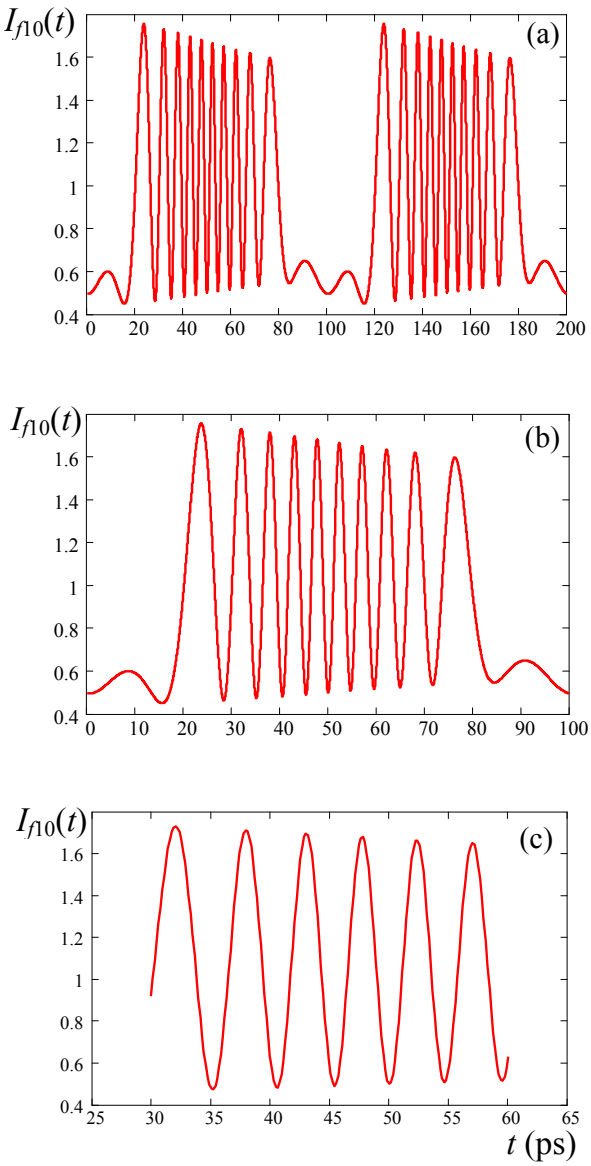

FIG. 7: (a) Time dependence of intensity of the radiation field filtered through the atomic vapor with the optical depth $\alpha_0 L = 453$. The spectral component $n = 10$ is tuned in resonance with the filter and the phase modulation index is $m_{10} = 11.8$. (b) The content of a bunch of pulses. (c) Zoom in a central part of the bunch. Other parameters and notations are the same as in Fig. 5

produced by phase modulation with the optimal value of the modulation index $m_{10} = 11.8$ (which is close to 2π). Such a filtering is capable to produce pulses as short as 2.5 ps, which are grouped in bunches consisting of 10 pulses. Thus, by phase modulation technique and subsequent filtering it is possible to create pulses whose duration is 40 times shorter than the modulation period.

V. FEMTOSECOND PULSES

It is also possible to create femtosecond pulses if high order harmonic of the frequency comb is removed. For simplicity we consider an ensemble of two-level atoms, for example, a vapor of alkaline atoms. We take the modulation frequency of the radiation field equal to $\Omega/2\pi = 10$ GHz. If the modulation index is $m_{100} = 104$, the amplitude of the frequency component $\omega_r - 100\Omega$ takes its first maximum value, which is proportional to $J_{100}(104) = 0.144$. Removal of this spectral component of the comb by atomic filter leads to high frequency oscillations of the transmitted field intensity. Duration of the pulses is estimated as $T_{EO}/400 = 250$ fs, where T_{EO} is the phase modulation period. The necessary modulation index $m_{100} = 104$ corresponds to a voltage, which is 33 times larger than the have-wave voltage of electro-optical modulator. One can expect to reach this value by increasing the voltage and/or the physical length of the modulator by an order of magnitude. This is technically possible if one employs electro-optical modulators, based on the integrated-optical waveguides guiding the light along a determined path analogue to optical fiber. Such modulators are fabricated in planar waveguiding substrates, which are 3 to 9 microns in width and depth (see, for example, specifications of JENOPTIK integrated-optical modulators). Therefore, half-wave voltage is quite low and maximum applicable voltage, which is approximately ± 30 V, produces phase shift 20π for red light. Extending three times the length of the electrodes, placed along the waveguide, and applying voltage ± 15 V, one can reach the desirable modulation index.

Below, the simplified picture of the filtering of high order harmonics of the frequency comb is illustrated. To avoid complicated expressions, we approximate the coherently scattered component of the field in Eq. (6) as proportional to $S_{100} = J_{100}(m_{100})$, assuming that the resonant absorption is close to 100%, i.e., $T_D(0) \approx 0$. We disregard the dispersive contribution of the neighboring spectral components. Time dependence of the intensity of the filtered radiation field is shown in Fig. 8 (a and c). The contrast of the pulses is not as large as for filtering of low order harmonics. This is because the intensity of the maxima and minima of the pulses can be estimated as $I_{\max} \approx [1 + 2J_{100}(104)]I_0$ and $I_{\min} \approx [1 - 2J_{100}(104)]I_0$. Since $J_{100}(104) = 0.144$, the maxima exceeds 30% the level of the intensity of the incident radiation field, while minima are smaller than I_0 on 30% of its value.

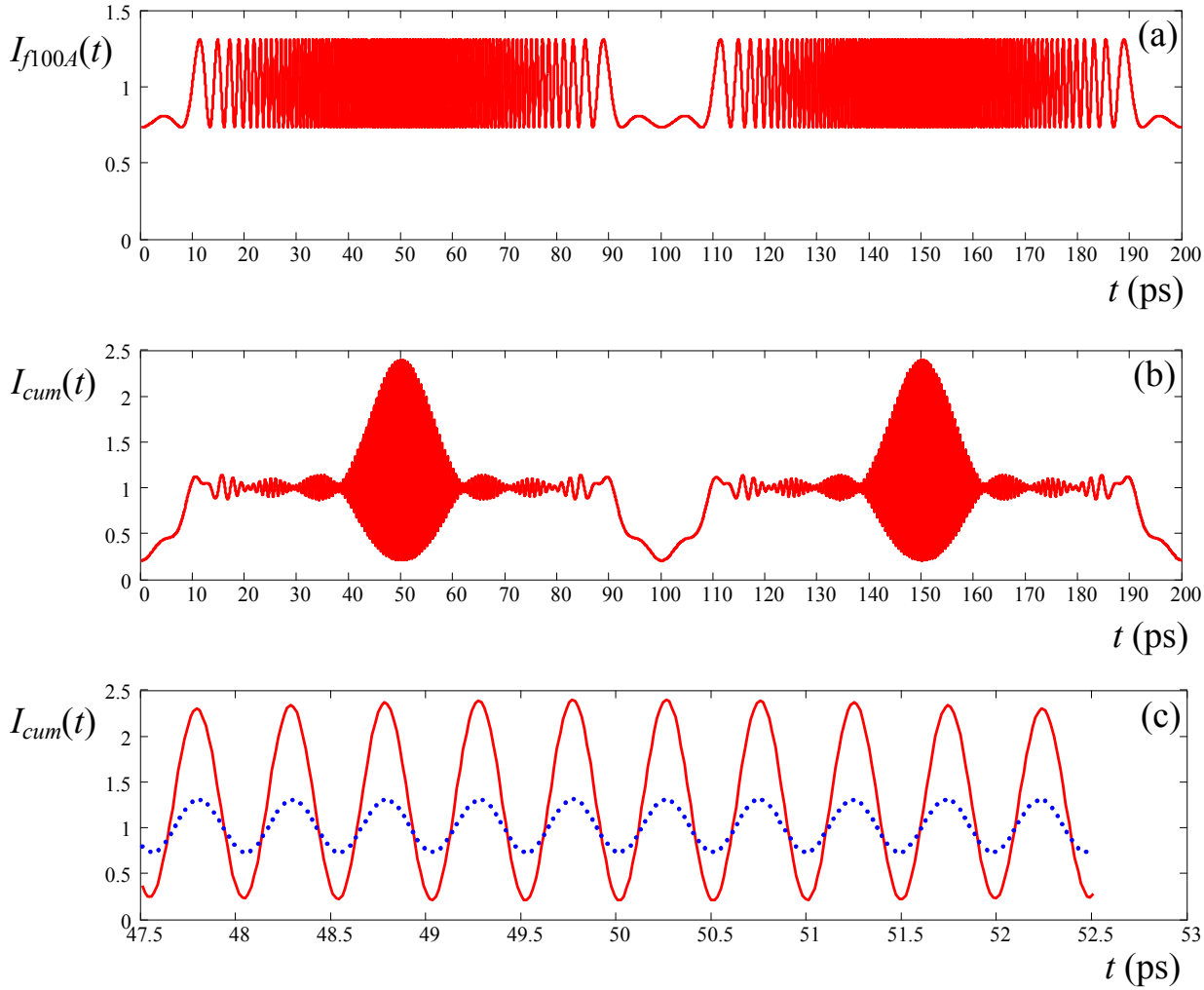

FIG. 8: (a) Time dependence of the intensity of the radiation field whose $n = 100$ spectral component is removed by the filter. The modulation frequency is $\Omega/2\pi = 10$ GHz. (b) Accumulative result of the filtering of the additional four spectral components of the frequency comb (see the text for details). (c) Zoom in a central part of the bunch. Accumulative effect is shown by solid line (in read), while dotted line (in blue) demonstrates the result of the filtering of only one spectral component with $n = 100$. Full width at half-maximum of the pulses is close to 250 fs as it is expected from simple estimations.

The amplitudes of high harmonics of the frequency comb decrease with increase of their number n since the first maxima of the Bessel functions $J_n(m_n)$ decrease with increase of the order n. To achieve large contrast of the pulses, one can remove several spectral components of the comb neighboring $n = 100$, if they have the same phase at time of the

pulse formation. As an example we consider the case if filtering of the comb component $n = 100$ is accompanied by filtering of the components with the numbers $n + 2k$, where k takes values ± 1 and ± 2. These spectral components have the same phase as $n = 100$ spectral component at times when central pulses of the bunch are formed. The amplitude of the filtered radiation filed with cumulative contribution of extra four spectral components of the comb due to their removal by separate filters is approximated as

$$E_{cum}(t) = E(t) \left[e^{im \sin \Omega t} - \sum_{k=-2}^{2} J_{n+2k}(m) e^{i(n+2k)\Omega t} \right], \tag{22}$$

where, for simplicity, it is supposed that $T_D(0) = 0$. The intensity of the filtered radiation field $I_{cum}(t) = |E_{cum}(t)|^2$ is shown in Fig. 8 (b and c). The central part of the pulse bunches demonstrates good contrast. The maximum accumulative effect of five spectral components is achieved if $m = 103$.

Cumulative filtering is possible if we have narrow bandwidth filters properly adjusted for chosen spectral components of the frequency comb. By a set of cells with atomic vapors it is hard to construct such a specific multifrequency filter having a large spacing between the absorptive frequency components of the order of 20 GHz. However, for example, cesium atoms have a large hyperfine splitting of the ground state seen as a spectral doublet with a spacing equal to 9.2 GHz. If we reduce the modulation frequency of the field phase down to $\Omega/2\pi = 4.6$ GHz, populate properly ground state levels by pumping Cs atoms such that both components of the doublet are strongly absorptive, then we could remove two spectral components of the frequency comb. An example of cumulative filtering of two spectral components 100Ω and 98Ω by cesium atoms is shown in Fig. 9 (b and c). The increase of the intensity of the pulses is clearly seen. Since the modulation frequency is decreased by a factor of two, the duration of pulses increases by the same factor to 500 fs.

Cumulative filtering of two spectral components by atoms with the spectral doublet produces pulses with modest contrast. The ratio of the maxima of the pulse intensities to their minima is close to three. To make pulse minima close to zero one can use destructive interference of the filtered radiation field $E_{cum}(t)$ with the field $E_{EO}(t)$ from the same electro-optic modulator but with the opposite phase and reduced amplitude, i.e.,

$$E_{rdc}(t) = E_{cum}(t) - RE_{EO}(t), \tag{23}$$

where R is the amplitude reduction factor. For the case of the filtering with the doublet

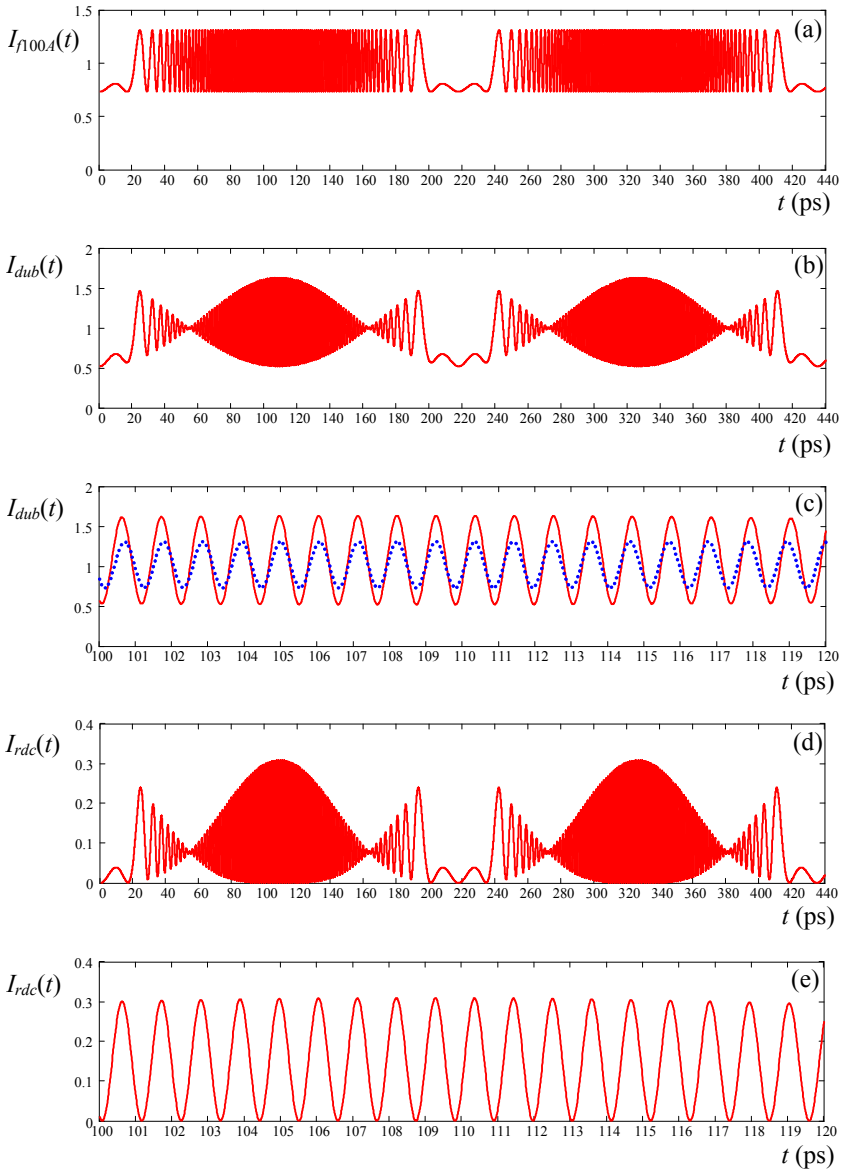

FIG. 9: (a) Time dependence of the intensity of the radiation field whose $n = 100$ spectral component is removed by the filter. The modulation frequency is $\Omega/2\pi = 4.6$ GHz and modulation index is $m_{100} = 104$. (b) Accumulative result of the filtering of the additional spectral component $n = 98$ of the frequency comb by the doublet (see the text for details). The modulation index is $m = 103$. (c) Zoom in a central part of the bunch. Accumulative effect is shown by solid line (in read), while dotted line (in blue) demonstrates the result of the filtering of only one spectral component with $n = 100$. Full width at half-maximum of the pulses is close to 500 fs as it expected from simple estimations. (d) The result of the filtering with the doublet, followed by reduction of the field amplitude due to the destructive interference with the reduced field from electro-optic modulator whose phase is shifted by π. (e) Zoom in the central part of the pulse bunch, shown in (d).

the reduction factor is $R = 1 - J_{100}(103) - J_{98}(103)$. Then, due the interference the pulse minima at the bunch center become zero, while the amplitude of the pulse maxima reduces to the value $2[J_{100}(103) - J_{98}(103)]E_0 = 0.55E_0$. The result of this interference is shown in Fig. 9 (d and e) for the field intensity $I_{rdc} = |E_{rdc}(t)|^2$.

Cumulative filtering is better to implement by organic molecules doped in polymer matrix, which undergo persistent spectral hole burning at liquid helium temperature. In such a filter the frequency resolution is limited by the width of the homogeneous zero-phonon lines of the chromophore molecules. For example, waveguide narrowband optical filter, which consists a planar waveguide with a thin polymer film containing molecules, which undergo persistent spectral hole burning at liquid helium temperature, demonstrates transmission bandwidth less than 1 GHz [42, 43]. Saturated holes are burned in waveguide geometry by illumination in the transverse direction with low absorption, whereas the probing is carried out in longitudinal wave guiding directions with high absorption. The waveguide with spectral hole burning can act as integrated sub-gigahertz narrow-band filter, which is proposed to observe slow light phenomenon [44, 45].

Comb structures of arbitrary shapes in transmission spectra were created experimentally in organic molecules doped in a polymer [46, 47]. Therefore, one can expect that it is experimentally possible to create a broad hole with, say, five absorptive peaks in it. For cumulative filtering it is enough to create a broad hole with the width ~ 10 cm^{-1} and five absorptive peaks in it with 20 GHz spacing and widths less than 1 GHz.

VI. DISCUSSION

Resonant filtering by cold atoms is capable to produce nanosecond and picosecond pulses in a controllable way. Nanosecond range could be achieved since cold-atom absorption lines are homogeneously broadened and hence very narrow. This filtering could be applied to create time bin qubits from single photon wave packets with a long coherence length. Several examples of such a qubit formation are experimentally demonstrated in Refs. [20, 21], where gamma-photons with coherence time 141 ns are transformed into a train of short pulses.

The absorption lines of the atomic vapors are Doppler broadened and they are relatively broad. Since the frequency spacing of the filtered combs must be larger than the absorption linewidth $\Delta\omega_D$, the vapor filtering is capable to produce picosecond or shorter pulses.

Nanosecond pulses cannot be produced by the vapor filters because of large absorption linewidth.

Both, cold and hot atomic filters are quite flexible in producing large variety of pulse trains. By changing the modulation index or frequency one can change the pulse duration and repetition time in a wide range. These filters should not be very thick and can have a fixed density, which is not constrained by a desired pulse duration.

This is very different from dispersive filters [6–10], which involve frequency modulation followed by dispersive compensator for producing short optical pulses. The idea of dispersive filtering have been based on the concept of chirp radar [49–51]. In the chirp radar system the transmitted pulse is of relatively long duration Δt during which time the instantaneous frequency is swept over the range $\omega \rightarrow \omega + \Delta_{chirp}$ satisfying the condition $\Delta_{chirp}\Delta t \gg 1$. The return pulse is passed through a dispersive network providing a differential delay Δt over the frequency range Δ_{chirp}. As a result, the energy at the beginning of the pulse is delayed so as to reach the end of the network at the same time as the energy at the end of the pulse. The duration of the compressed pulse produced this way is of the order of $\delta t \approx 1/\Delta_{chirp} \ll \Delta t$.

In optical domain electrooptical modulation of the radiation frequency [7] spreads the field spectrum over the range $\omega_r - m\Omega \rightarrow \omega_r + m\Omega$. In this scheme dispersive properties of atoms are used to create frequency-dispersive group velocities of the spectral components produced by modulation. Therefore, dispersive filtering scheme requires resonant media with a very large optical thickness and employ large offset $\Delta = \omega_r - \omega_f$ between the radiation frequency ω_r and the absorption line center ω_f ($|\Delta| > m\Omega + \Delta\omega_D$) to avoid the absorptive losses. These constrains force to work with relatively small frequencies Ω and large modulation index m. Then, many spectral components of the comb acquire appreciable dispersive phase shifts with large difference between blue and red borders of the comb spectrum. The disadvantage of the dispersive filter is a requirement of a fixed, certain value of the vapor density, which is necessary for pulse compression for a given values of the modulation index m and frequency Ω, and offset between the radiation frequency and the absorption line center Δ. Any deviation from the predefined, fixed value of the vapor density essentially reduces pulse production efficiency.

VII. CONCLUSION

The resonant method of converting phase modulation into amplitude modulation of the radiation field in optical domain is discussed. Phase modulation could be implemented by electro-optic modulator, which converts a single line radiation field into a frequency comb consisting of fundamental frequency ω_r and sidebands spaced apart at distances that are multiples of the modulation frequency, i.e., $\omega_r \pm n\Omega$ ($n = 1, 2, ...$). The intensity of the spectral components of the comb is proportional to $J_n^2(m)$, where m is the phase modulation index. If the n-th spectral component of the comb is tuned in resonance with atoms in the filter whose resonant absorption line is narrower than the frequency spacing Ω of the comb, then the filtered field is transformed into pulses demonstrating a conversion of the phase modulation into intensity modulation. The effect is explained by the interference of the coherently scattered resonant component in the forward direction with the whole comb. Constructive interference of the fields results in formation of pulses. Their destructive interference is seen as dark windows. Three examples of the resonant filter are analyzed. They are an ensemble of cold atoms, atomic vapor of alkaline atoms, and organic molecules doped in polymer matrix. It is shown that it is preferable to work with the filters with moderate optical depth. If the optical depth is large, then due to dispersive wings of the filter many spectral components of the comb are modified producing pulse corruption. Creation of nanosecond, subnanosecond, and femtosecond pulses is analyzed. The number of pulses and their spacing is well controlled by the modulation frequency and modulation index. The proposed method could be also applied for photon shaping. Single photons of spectral widths ranging from several MHz up to 1 GHz could be modified to encode the information into time-bin qubits.

VIII. ACKNOWLEDGMENTS

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