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Commutation of Projections and Characterization of Traces on von Neumann Algebras. III

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Abstract We obtain new necessary and sufficient commutation conditions for nonnegative operators and projections in terms of operator inequalities. It is shown that in the general case in this inequalities the projections cannot be replaced by arbitrary nonnegative operators with preservation of operators commutativity. We also present new necessary and sufficient commutation conditions for projections in terms of operator inequalities. These inequalities are applied for trace characterization on von Neumann algebras in the class of all positive normal functionals. We also consider the following problems: I. Characterization of traces among arbitrary weights on von Neumann algebras. II. Characterization of tracial functionals among all positive linear functionals on C^*-algebras. III. Characterization of commutativity for C^*-algebras.

Keywords Hilbert space · Linear bounded operator · Von Neumann algebra · Spectral theorem · C^*-algebra · Weight · Trace · Normal functional · Operator inequality · Projection · Commutativity of operators

1 Introduction

The present paper is a sequel to [5, 6]; we use the notation and terminology from these works. In [5, 6] we established some new criteria for the commutation of projections in terms of operator inequalities. These inequalities were applied for characterization of traces among all positive normal functionals on von Neumann algebras.

Dedicated to memory of Professor Peter Mittelstaedt

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