Vacuum polarization of a quantized scalar field in the thermal state in a long throat

A. A. Popov

Institute of Mathematics and Mechanics, Kazan Federal University,
Kazan, 420008, Russia
E-mail: apopov@ksu.ru

Vacuum polarization of scalar fields in the background of a long throat is investigated. The field is assumed to be both massive or massless, with arbitrary coupling to the scalar curvature, and in a thermal state at an arbitrary temperature. It is shown that $\langle \phi^2 \rangle$ does not depend on the temperature of thermal state.

Keywords: Vacuum polarization; scalar field; wormhole.

The study of vacuum polarization effects in strong gravitational fields is a pertinent issue since such effects may play a role in the cosmological scenario and the construction of a self-consistent model of black hole evaporation. These effects may be taken into account by solving the semiclassical backreaction equations,

$$G^\mu_\nu = 8\pi \langle T^\mu_\nu \rangle, \tag{1}$$

where $\langle T^\mu_\nu \rangle$ is the expectation value of the stress-energy tensor operator for the quantized fields.

The main difficulty in the theory of semiclassical gravity is that the effects of the quantized gravitational field are ignored. The popular solution for this problem is to justify ignoring the gravitational contribution by working in the limit of a large number of fields, in which the gravitational contribution is negligible. Another problem is that the vacuum polarization effects are determined by the topological and geometrical properties of spacetime as a whole or by the choice of quantum state in which the expectation values are taken. It means that the calculation of the functional dependence of $\langle T^\mu_\nu \rangle_{ren}$ on the metric tensor in an arbitrary spacetime presents formidable difficulty. Only in some spacetimes with high degrees of symmetry for the conformally invariant fields $\langle T_{\mu\nu} \rangle_{ren}$ can be computed and equations (1) can be solved exactly[1–5]. Let us stress that the single parameter of length dimensionality in such a problem is the Planck length l_{Pl}. This implies that the characteristic scale l of the spacetime curvature (which correspond to the solution of equations (1)) can differ from l_{Pl} only if there is a large dimensionless parameter. As an example of such a parameter one can consider a number of fields, the polarization of which is a source of spacetime curvature [a]. For the quantized scalar field such a parameter can be the coupling constant ξ of scalar field to the curvature of spacetime. In the case of $|\xi| \gg 1, \xi < 0$ the vacuum fluctuations of a quantized scalar field can determine the curvature of spacetime so-called *a long throat*[6].

In this work we have shown that in a long throat the effect of vacuum polarization of a quantized scalar field in the thermal state does not depend on temperature and conditions at infinity. This implies that in considered situation $\langle\varphi^2\rangle$ is a local quantity for any finite mass m of the quantized field, including $m = 0$.

Acknowledgments

This work was supported in part by grant 13-02-00757 from the Russian Foundation for Basic Research.

References

1. A. Starobinsky, *Phys. Lett. B* **91**, 99 (1980).
2. S. Mamayev and V. Mostepanenko, *Sov. Phys.–JETP* **78**, 20 (1980).
3. L. Kofman, V. Sahni and A. Starobinsky, *Sov. Phys.–JETP* **85**, 1876 (1983).
4. L. Kofman, V. Sahni, *Phys. Lett. B* **127**, 197 (1983).
5. L. Kofman, V. Sahni, *Phys. Lett. A* **117**, 275 (1986).
6. A.A. Popov, *Class. Quantum Grav.* **22**, 5223 (2005).

[a]Here and below it is assumed, of course, that the characteristic scale of change of the background gravitational field is sufficiently greater than l_{Pl} so that the very notion of a classical spacetime still has some meaning.